# Three Essays on Fiscal Policy and Government Debt

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## Introduction

Fiscal policy and government debt are topics that concern millions of citizens at the same time. For example, increases in income taxation or cuts in government expenditures affect virtually every citizen in a country in one way or another. If a government accumulates an unsustainable debt burden, the citizens are the ones who will finally have to bear the consequences in the form of higher taxes or lower public expenditures. Therefore it should not be surprising that fiscal policy and government debt have been hotly debated among economic researchers and practitioners since the beginning of economic thought. The three essays of this doctoral thesis aim at contributing to this classical debate.

How much government debt should a benevolent government issue? How does restrained mobility affect income sorting? How is the level of government debt determined politically by young and old voters? From the myriad of possible questions concerning fiscal policy and government debt, these three questions were selected as topics for the three essays of this doctoral thesis:

The first essay with the title *Wealth Inequality and the Optimal Level of Government Debt*, joint work with Christoph Winter, is concerned with the question of what the optimal level of government debt should be. Government debt has become a major concern in the aftermath of the financial crisis of 2008-2009. Most non-economists (or even economists) probably think of government debt as something negative, potentially constraining the scope of action of a government. This view would suggest avoiding a long-run stock of government debt. Some authors, however, have found a positive role of government debt or bonds, as an asset providing a means for saving and thus liquidity to the financial market (see, for example, Woodford, 1990 or Aiyagari and McGrattan, 1998). But what if many households don't even own bonds and therefore do not really benefit from this positive effect? The Survey of Consumer Finances, for instance, shows for the United States (US) that 20% of households own no wealth or are even indebted. Furthermore, a negative effect of government debt when household's borrowing is constrained, besides being a restriction on future fiscal policy, is to crowd out firms that are searching for funds to borrow. Considering these aspects, it is an interesting quantitative question whether the negative or the positive effects of government debt on the economy and the economic well-being or *welfare* of households will prevail. To assess this question, it is important to take into account the high inequality of wealth between households and the fact that some households own no bonds or assets or are even indebted. This motivates our reassessment of the quantitative question of what the optimal level of government debt should be.

Whereas the first essay asks a *normative* question by analyzing the optimal level of government debt from a Utilitarian perspective, the second and third essays provide a *positive* analysis. Here, the goal is not to define an *optimal* or *ideal* state of the world, but to improve our understanding of how the economic and political decisions of agents are formed and lead to certain outcomes.

The second essay *Homeownership, Mobility and Local Income Redistribution*, joint work with David Stadelmann, is concerned with the question how restrained mobility affects income sorting. An income sorting equilibrium arises when households sort themselves into communities according to income classes. Taxes and house prices are endogenous variables influencing income -orting, but are also influenced by it themselves. A typical result is that the richest households live in the community with the lowest tax rate and the highest house prices, the second-richest households in the community with the second-lowest tax rate and second-highest house prices, and so on. It is important to study such income sorting as it represents an important aspect of daily lives of citizens and it determines how much income redistribution is possible at the local level. Therefore, various authors have studied the underlying factors for income sorting to arise. For example, Hansen and Kessler (2001) analyze the influence of community sizes or Kessler and Lülfesmann (2005) and Schmidheiny (2006) the influence of heterogeneity in preferences for public goods for the existence of income sorting equilibria. Our contribution in this essay is to analyze the influence of households with different mobility costs. For this purpose, we use a stylized set-up where we differentiate between two types of households with respect to their mobility costs: *homeowners* with infinite mobility costs and *renters* with zero mobility costs. The idea is that homeowners are settled down definitely in a specific community or, in other words, they do not move anymore. Consequently, they cannot evade high taxes. In contrast, renters can react to incentives created by different tax rates in different communities by moving to another community. Introducing those two groups in a model of endogenous policy determination, allows to address the question in what way, theoretically, homeownership or mobility of households can affect income sorting.

The third essay *Public Debt in a Political Economy* investigates the role of different factors for the determination of the level of government debt if government debt is viewed as the outcome of a political process. In contrast to the second essay, which is concerned with local policies constrained by potential competition between jurisdictions, the third essay abstracts from these considerations by focussing on national fiscal policies of public good provision, income taxation and government debt. But similar to the second essay, the third essay incorporates the idea that fiscal policies are endogenous, arising from the choice of voters in democratic societies. The political institution of repeated elections is an important determinant of the level of government debt. If the government is reelected, government debt provides a link between those different legislative periods, permitting one generation to exert an influence on the next generation by setting a specific level of government debt. Among the first to note this potential strategic behavior of governments over time were Alesina and Tabellini (1990) and Persson and Svensson (1989). In the spirit of this earlier work, this essay aims at providing a first step in the direction of a theory of the determination of government debt in a framework with young and old voters and a national bond market. The work is inspired by Song et al. (2011), who analyze a similar framework with a financial market that is open internationally and where it is assumed that the international market is much bigger than the local market. It is a well-known puzzle of international macroeconomics that savings and investment are strongly correlated despite open financial markets (so called "home bias"). Real-world financial markets therefore seem to be best characterized as being between the two extremes of completely open and completely closed. This is the reason for addressing the question of the determination of government debt for the case of a closed financial market as well.

Although the three essays of this doctoral thesis treat three clearly distinct topics (optimal government debt, local income sorting and the political economy of government debt) with different goals (normative and positive) and using very different modeling tools, there are nevertheless some conceptual similarities. One similarity is the consideration of some heterogeneity between economic agents. In fact, although concerning virtually everyone to some degree, fiscal policy and government debt often concern some citizens more than others. A rich person will be more concerned about the price of a bond than a poor person who cannot afford to buy any bonds. A homeowner who plans to stay in a community for the rest of his life is affected by changes in local income taxation differently than a renter who is only temporarily living there with the possibility of leaving at any moment. A young voter is differently concerned about a high level of government debt than an old voter, because he will most probably still live to experience the consequences of this burden on fiscal policy in his or her country. A common denominator of the three essays is thus that they deviate from the assumption of a "representative agent", by allowing for some form of heterogeneity among economic agents: rich and poor, homeowners and renters, young and old.

A further common denominator in all three essays is the importance of general equilibrium effects and/or strategic effects. In the first essay, whereas fiscal policy is taken as exogenous, general equilibrium effects play a role because of the endogeneity of demand and supply in different markets (labor market, goods market, capital market). Most importantly, we argue in this essay that an increase in government debt leads to a higher increase in the supply of assets than in the demand for assets because some agents are borrowing-constrained. A different endogeneity exists for the kind of location and voting equilibrium that is analyzed in the second essay. Here fiscal policy itself is endogenous as voters decide by majority voting how much taxes and redistribution they want. In the third essay as well, fiscal policy is endogenous. General equilibrium effects play a role in that consumption and savings decisions are affected by policy choices. Strategic effects play a role in that the voters of this generation will consider the effect that their decision on government debt can have on the next generation of voters.

In the following, I will provide a short non-technical summary of each of the essays of this doctoral thesis. The complete working papers are in the Appendix.

## Essay 1: Wealth Inequality and the Optimal Level of Government Debt

In this essay, Christoph Winter and I ask the quantitative question of how much government debt a benevolent government should issue. As a framework for our analysis, we use a model that often serves as a 'workhorse' of applied quantitative macroeconomics: an incomplete market model as in Aiyagari (1994) (see, for example, Ljungqvist and Sargent, 2004, Chapter 17, for a discussion of the theory and various applications of the incomplete market models). Insurance markets are absent and households can only self-insure against fluctuations in their personal labor income by trading a risk-free one-period bond, subject to a short-selling constraint. The idea is that some risks to your personal labor income (or equivalently to your individual labor productivity) are not or only partly insurable, such as health, divorce or injuries. What makes this framework so interesting from a modeling perspective is its ability to generate an equilibrium with a stationary wealth distribution that is very similar to empirical wealth distributions. It thus allows us to set up the model so as to match the empirical data quite closely, a process usually called *calibration*. As a means of illustration and to be comparable with previous research in this area, the focus is on the US.

A seminal contribution for this kind of quantitative work on the economic welfare effects of government debt is the article by Aiyagari and McGrattan (1998) in which they argue that the optimal level of government debt under incomplete markets is positive. In a version of the model also calibrated to the US economy, they find that the optimal long-run level of government debt relative to the gross domestic product (GDP) is 0.6. This value is very close to the ratio of government debt to GDP that was observable in the

US in the last decades before the 2008-2009 financial crisis (approximately 2/3). Hence, they conclude that government debt is already close to the optimal level. Furthermore, they find small economic welfare effects of a change in government debt. These findings suggest that it is not necessary to take policy actions concerning the quantity of government debt from a macroeconomic perspective, or in other words, even large quantities of government debt are not such a bad thing. Aiyagari and McGrattan (1998) explain this possibly surprising finding by hinting at the fact that issuing government debt crowds out private capital, which can be welfare improving in an incomplete-markets setting. This is because a lower capital stock is associated with a higher interest rate, which facilitates self-insurance by increasing the return of the risk-free bond.

An important contribution of our essay is to show that the optimal quantity of debt that results in an incomplete markets model depends crucially on the degree of wealth heterogeneity. In particular, we show that once we take into account the high inequality of earnings and wealth that exists in the US economy, the optimal level of government debt becomes negative, i.e., the government should accumulate assets, not debt. This finding is quite intuitive: the actual wealth distribution implies that a big fraction of the population holds almost no assets (or is even in debt); so those households receive income only from labor earnings. Since government debt crowds out private capital, this implies a lower marginal product of labor and thus a lower wage rate. Consequently, government debt adversely affects income of the wealth-poor, who account for a big fraction of the population. This the reason why a benevolent government should accumulate capital as well, accepting a lower capital income of the savers (resulting from a lower interest rate) in favor of a higher wage rate.

Although higher than found by the earlier literature (notably, by Aiyagari and McGrattan, 1998; Flodén, 2001), the overall economic welfare consequences of government debt are found to be still moderate. This picture changes however when looking explicitly at the economic welfare of different wealth groups: the wealth-poor, with no or negative assets, the wealth-rich, who as a group hold 70% of all assets, and the middle class, the group in-between. Interestingly, the wealth-poor benefit the most from the decrease proposed as an optimal policy, and the effects are much larger than the overall or average welfare effect. Intuitively, the wealth-poor are part of the group that benefits most from high wages and low interest rates, as they have a lot of labor income and are sometimes indebted. The downside of low government debt is that the low interest rate discourages savings, and many (unlucky) previous middle-class households become wealth-poor.

Until now we have focussed on the long-run stock of debt and level of taxes without considering how a transitional period can lead the economy to those "steady-state" situations. Given that we found a lower long run stock of debt improves economic welfare, the question is now, whether or how this desirable situation can be achieved? Clearly, to lower the level of government debt the government would have to increase taxes or reduce expenditures. In this essay, the focus is on the policy of increasing taxes. More precisely, in a transitional analysis, we balance short-run welfare losses of increased short-run taxation against the long-run gains of a reduced level of government debt. Even considering those transitional effects, it is still possible to achieve overall economic welfare gains by reducing government debt, although they are now somewhat lower.

Another interesting aspect we analyzed in more detail is the crowding out of the private debt market by public debt. In an extension, we analyze the influence of government debt on the private debt market by assuming endogenous borrowing limits. More precisely, we assume that households can lend to and borrow from each other, but they could also potentially default on their financial liabilities. Upon default, households would be excluded from future borrowing and lending forever. As in the recent articles by Zhang (1997) and Ábrahám and Cárceles-Poveda (2010), an individual household will thus be able to borrow only as much as a potential lender will be ready to give without having to fear a default. Formally, there is an *endogenous limit* to borrowing, set in such a way that households are indifferent between defaulting and participating in financial markets. In fact, as it is in every borrower's own interest to observe this limit, there will be no default in equilibrium.

The effect of a reduction in government debt (accumulation of government assets) under these conditions is the following: if the government accumulates assets, the resulting fall in the interest rate makes default less attractive, as a lower interest rate makes it easier for households to service their debt. Consequently, borrowing limits become looser, and the fraction of the population that is in debt increases. We find that this effect causes the optimal long-run level of government assets to be even higher compared to the case when borrowing limits are exogenous.

## Essay 2: Homeownership, Mobility and Local Income Redistribution

Due to the long-term perspective implied when establishing a *home* and due to higher transaction costs, homeowners are less mobile compared to renters. This essay analyzes theoretically how homeownership or mobility affects local income sorting equilibria. More precisely, we consider a framework of locational choice on the one hand, and endogenous policy determination through voting, on the other hand (see among others Hansen and Kessler, 2001; Kessler and Lülfesmann, 2005; Schmidheiny, 2006). Such a set-up corresponds especially well to a federalistic and democratic country like the US or Switzerland. In such a framework, usually households of a specific region, i.e. a region with several separate jurisdictions, are characterized by income differences. In consequence, part of the population is the net beneficiary of a redistributive policy and the other part the net contributor. For this reason, there is an incentive for net contributors to move away from a high-tax jurisdiction to a low-tax jurisdiction and vice versa for net beneficiaries.

We model the household decisions in two stages: A household first decides in which jurisdiction it wants to settle, considering expectations about the amount of redistribution that will be implemented in this jurisdiction. Second, each household has the possibility of electing a local political candidate with a certain electoral program of redistributive taxation through majority voting. Majority voting implies that any political candidate proposes the policy which at least 50% of the electorate support (the pivotal person being the so called "median voter"). A well-known theorem from the political economy literature therefore states that (under certain conditions on the preferences which are satisfied in our framework) the policy outcome under majority voting is always the one preferred by this median voter (median voter theorem). When households differ in terms of income, it clearly depends on the income distribution among the households and the resulting inequality, how much redistribution the median voter will prefer and, hence, how much redistribution will result in equilibrium. If inequality is high among households in a jurisdiction, which means a lot of comparatively poor households and only a few relatively rich ones, the median voter is poor and will thus prefer a high amount of redistribution. If inequality is low, which means that everyone has approximately the same income, the median voter is almost as rich as anyone else and consequently will prefer low (or, in the extreme case where everyone is equally rich, no) redistribution. Of course, all this can already be foreseen by a rational household deciding where to live in the first place. Thus, in equilibrium every household finds his or her expectations fulfilled and doesn't want to move anymore.

Goodspeed (1986) was among the first to show for models with local income redistribution that a typical equilibrium outcome of such a model is that households sort themselves according to income classes, the richest households living in the jurisdiction with the lowest tax rate, the second-richest in the one with the second-lowest tax rate, and so on. This sorting result means that redistribution is less extreme than if it were implemented at the national or regional level, because redistribution can then happen only inside each income class and not between the classes.

The contribution of our essay is to consider the role of homeownership or mobility for income sorting equilibria of this kind. We begin by presenting some empirical facts about income sorting and homeownership on which to base our analysis. For instance, we present data for Swiss cantons suggesting a non-linear relationship between tax differences between communities (which is a quantitative measure of income sorting within a canton) and the homeownership rate. Then we present our model of local income redistribution. We model the redistributive policy, as is standard in the literature, as a transfer financed by distortive income taxation. Thus, higher taxes mean more redistribution. To emphasize the role of mobility, we assume that homeowners and renters differ strongly with respect to their mobility, in kind of an extreme way: homeowners are completely immobile, whereas renters are perfectly mobile. If part of the population is not mobile, this changes the amount of possible redistribution relative to the case where everyone is mobile, because those additional immobile households change the voting outcome by their presence. Or put differently: the median voter might be another person, if part of the population is not mobile. How exactly redistribution is changed, of course, depends on how much income each homeowner earns. We assume that the income distribution is the same for renters and homeowners, which we view as the most natural and neutral assumption to begin with. Furthermore, to illustrate the general mechanism we want to emphasize, we concentrate on the case of two jurisdictions.

Our analysis suggests the following results: First, if there are no or only a few homeowners, there is no income sorting equilibrium. The reason is that the income distribution is very skewed, meaning that there are clearly fewer super rich persons than extremely poor persons. Suppose for simplicity everyone is a renter. In this case, if there was income sorting, the poor and the middle class would be in one jurisdiction, whereas the moderately rich and the super rich would be in the other jurisdiction. Where will the tax rate be higher? The answer is: in the rich jurisdiction. The reason is that inequality is much higher between moderately rich and super rich than between poor and middle class. Thus, in such a situation, the richest households of the rich jurisdiction would start moving to the other jurisdiction trying to evade taxes. But this in turn would increase inequality in that jurisdiction, which means more redistribution and a higher tax rate. The rich would move again, and so on. There is no "equilibrium", because the process never stops.

Second, at some point there is a threshold value for the homeownership rate such that for rates higher than this value there exists a sorting equilibrium of the kind described above. The reason is that the presence of homeowners has a dampening effect on inequality in the rich jurisdiction, which permits the rich to agree on a tax rate that is somewhat lower than the tax rate in the poor jurisdiction. For a range of moderate homeownership rates, the difference between tax rates increases with the homeownership rate. Intuitively, as long as the inequality between homeowners is lower than the inequality between the richest renters, a higher presence of homeowners decreases the equilibrium tax rate in the rich jurisdiction, whereas the tax rate in the poor jurisdiction is less affected.

Third, there is a range of high homeownership rates for which the difference between tax rates decreases with the homeownership rate. Intuitively, from some point onwards the tax rate in the poor and the rich jurisdiction have to be more and more similar, as the high number of homeowners with an equilibrated income distribution makes the income distribution in both jurisdictions more and more similar. Finally, we find that an increase in overall regional inequality increases the likelihood of income sorting for each possible value of the homeownership rate.

## Essay 3: Public Debt in a Political Economy

How can it happen that relatively rich countries end up in an unsustainable situation of high indebtedness? What characteristics of a country lead to high indebtedness in theory? What role does the financial market openness of a country play? And finally, what institutional mechanisms are more prone to induce high government debt? The third essay, *Public Debt in a Political Economy*, aims at providing answers to those questions. The inability of democratic governments to commit to policies over long periods of time is viewed as crucial to an understanding of how the level of government debt is determined. A government that needs to be periodically re-elected cannot directly decide on what fiscal policy is implemented in the future. Thus, as already noted by Persson and Svensson (1989) and Alesina and Tabellini (1990), governments that are re-elected every legislative period may try to strategically influence decisions of the next period's government by setting a specific level of government debt.

More precisely, this essay investigates the political conflict between young and old voters and what it means for the determination of government debt (voters are referred to as *agents* of the model in the following). In economics, intergenerational questions are usually analyzed using models with overlapping generations (OLG). Two lifetime periods are sufficient to analyze the basic mechanism here. The first period stands for the working-life of an agent, and the second period is the retirement period. The economic difference between those periods lies in the labor income profile, all labor income accruing in the working-life period, whereas in the retirement period the agent has to live off his or her savings. The different generations of agents *overlap* in the sense that each time period is the working-life period for some *young* agents who newly entered the labor market and at the same time is the retirement period for some *old* agents. Therefore, at each point in time two types of agents simultaneously live together: the old and the young. In this model, I assume a lifetime period to be equal to a legislative period (simply referred to as *period* in the following).

Fiscal policy consists of public good provision (which benefits both types of agents), income taxation (concerning only the young working agents) and government bond issues. The bonds promise one unit of consumption next period. They will be used by the young agents to provide for their old age, as no other savings instrument is available. Importantly, the government has to meet a budget constraint, meaning that the income from taxation and new bond issues should provide for the expenditures on public goods and the units of consumption promised in the last period in the form of bonds.

This model is inspired by Song et al. (2011) with the important difference that a closed financial market is assumed. In their model, the internationally open financial market implies that the number of bonds the government issues has no influence on the price of bond, as the international market is very large compared to the national market. In contrast to earlier literature, Song et al. (2011) find as a result of their analysis that the lack of commitment does not lead to higher levels of government debt, but rather to lower ones. The reason is that young agents today are worried about their public good provision in the next period and thus prefer not to leave such a high debt burden to the next government. They will thus put a brake on financing too much expenditure via debt.

The contribution of this essay is to analyze a model with a closed financial market. In this case, every bond issued by the government has to be bought by a young agent ready to save for retirement. This means that the government can manipulate the interest rate via bond issues. If, for example, starting from a given number of bond issues, the government decides to increase the number of bond issues, the interest rate will increase because young agents must be persuaded to buy even more bonds (the law of demand and supply). If, on the contrary, the government decides to issue fewer bonds, the interest rate will decrease by an analogous mechanism as above. In such a closed economy model, government debt is thus used as a savings instrument for young agents today, but does not help the government to finance itself, at least in real terms. Intuitively, there is only a given amount of real resources in each period, and as there is no possibility to borrow abroad, it is not possible to change this amount of given resources. All that the government can do is to allocate more of those resources to government activities, such as public good provision, or repaying debt obligations, by increasing taxation. So what is the role of new bond issues then? The answer is that they matter for the next period, as they have to be paid back to the old retired population. In this way, government debt, although it cannot finance expenditures, provides a link between different periods.

As a benchmark for comparison, I first analyze the commitment solution, where the first generation decides on the whole future path of taxes, public goods and government debt. The old agents do not care about government debt at all, because it is not useful to finance public expenditures today. For young agents, the role of government debt issues is to achieve the optimal mix between private and public consumption in old age, with more bonds meaning relatively more private consumption (bond proceeds) and fewer bonds meaning relatively more public consumption.

Then in the second step, I assume that young and old voters can reelect the government every period. All voters of one type (young or old) have the same preferences regarding a policy proposal of a political candidate, but they differ idiosyncratically in terms of *ideology* with respect to this candidate. In the political economy literature, ideology summarizes everything that could be considered important for the voting decision concerning the political candidate, except for the policy question. It has been shown that under such conditions any political candidate, maximizing his or her probability of being elected, will consider a weighted sum of the preferences of the different types of agents, weighing them with their political power (for example, political power = share in the population). In the political equilibrium, it does not matter which candidate is elected as the candidates all propose the same policy.

But what will this policy look like more specifically? I show that the tax, public goods and debt issuing policy depend on how much debt there is or, in other words, how much consumption was promised to old agents. The higher the debt burden is, the higher the taxes will be and the lower the public goods. The new bond issues are used to determine the consumption of tomorrow's old agents on the one hand (similar as in the commitment case) and to influence tomorrow's decision-making strategically on the other hand. This strategic effect was not present, because not necessary, in the commitment case. The effect depends on whether public and private goods are substitutes or complements. The essay shows that although there are examples of both of the above, it is not clear from the empirical literature what would be a good aggregate assumption, since the results are contradictory. Therefore both cases are considered.

The strategic effect permits today's young voters to have an influence on the next generation's young voters. Young voters today know that the reaction of the next generation of voters to a high level of government debt will be to reduce public expenditures and increase taxes to be able to finance the debt burden. Young voters today thus deviate from the optimal tradeoff level of government debt to induce higher taxation of tomorrow's young generation and in this way rip them off their resources. However, the higher government debt level also induces cuts in public goods expenditures, so that the young today must substitute some public goods for private goods in their old age. The higher the substitutability of public and private goods, the stronger thus the strategic use of government debt and the higher the debt level.

Regarding the answer to the question of which institutional mechanisms are particularly prone to favoring high debt accumulation, this essay thus shows clearly that it depends on the substitutability of public and private goods. It turns out that if public and private goods are net substitutes, a commitment mechanism leads to a lower debt level than a voting mechanism. If, however, public and private goods are net complements, a voting mechanism leads to a lower debt level than the commitment mechanism.

Coming back to the question asked in the beginning as to how it can happen that relatively rich countries exhibit high government debt, this essay indicates the possible underlying causes. On the one hand, government debt partly constitutes the savings of the young generation for their old age. On the other hand, in a political conflict between the generations, government debt can also be used as a strategic instrument to influence the next generation, leading to potentially higher government debt (in the case of net substitutability between public and private goods). Both aspects can lead to a relatively high debt burden, even if a country is rich and developed.

Concerning the characteristics of a country that lead to high indebtedness in theory, this essay identifies, in the next step, the influence of underlying preferences on the level of government debt in a country: the preference for public goods, intergenerational altruism and the political power of the old. A higher preference for public goods and higher altruism lead to a lower level of government debt. The influence of voting power depends on the direction of the strategic effect, higher voting power leading to a debt level nearer to the commitment level.

Finally, the question of the role of financial market openness of a country is analyzed in this third essay, last part of the present doctoral thesis, by comparing the results to those of the small open economy set-up by Song et al. (2011). Without tax distortions, government debt is generally lower in the political equilibrium of closed economies. With high tax distortions, however, government debt can be lower in open economies.

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# Appendices

# Appendix A

# Wealth Inequality and the Optimal Level of Government Debt


## Paper Summary

In this paper, we quantitatively analyze to what extent a benevolent government should issue debt in a model where households are subject to idiosyncratic productivity shocks, insurance markets are missing and borrowing is restricted. In this environment, issuing government bonds facilitates saving for self-insurance.

Despite this, we find that in a calibrated version of the model that is consistent with the skewed wealth and earnings distribution observable in the U.S., the government should buy private bonds, and not issue public debt in the long run. The reason is that in the U.S., a large fraction of the population has almost no wealth or is even in debt. The wealth-poor, however, do not profit from an increase in the interest rate following an increase in public debt. Instead, they gain from higher wages that result from a reduction in debt.

## A.1 Introduction

In this paper, we ask the following question: what is the optimal level of government debt in a world where insurance markets are incomplete, households are thus subject to uninsurable shocks to their labor income and borrowing is restricted?

We consider a model in the spirit of Aiyagari (1994) where households are subject to idiosyncratic shocks to their labor productivity and insurance markets are missing. Only one-period risk-free bonds are available for households to self-insure against income shocks. Bonds are issued by firms (as claims to physical capital) or by the government in the form of public debt. In the absence of aggregate risk, claims to physical capital and public debt are perfect substitutes. As noted by Albanesi (2008), issuing government bonds in such an environment might be an effective way to improve risk-sharing and aggregate welfare. With the help of this model, we analyze quantitatively to what extent the U.S. government should issue debt. Perhaps surprisingly, we find that the optimal level of government debt is negative in the long run. In other words, our findings suggest that in a long run perspective the government should save and supply capital to the production sector instead of issuing bonds.

Our finding can be explained as follows. As it is well known from the seminal papers of Woodford (1990) and Aiyagari and McGrattan (1998) as well as more recently by (Gomes et al., 2008, 2010) issuing government bonds can have very different effects in an environment with incomplete markets compared to a setting where markets are complete. Heathcote (2005) makes a similar point with respect to fiscal policy in general. If borrowing constraints are binding, raising government debt crowds out private capital, even when taxes are lump-sum. In this case, households that face binding borrowing constraints will not increase their savings one-to-one in response to an increase in debt, and Ricardian Equivalence breaks down. This implies that the demand for private bonds will not meet the supply of private bonds issued by the firm. We say that public debt crowds out private capital, and therefore also production and output. As a result, the equilibrium interest rate that clears the private bond market will increase, and the marginal product of labor will decrease. Heathcote (2005), who studies tax reforms in a model with borrowing constraints, finds that this effect can be quantitatively important. Moreover, if taxation is distortionary instead of lump-sum, the negative effect of government debt on capital and output is even stronger, due to an inefficiently low supply of labor and capital. Clearly, both the crowding out of capital as well as the efficiency losses due to distortionary taxation reduce aggregate welfare.

In an world with incomplete markets, there are two additional effects how government debt can influence the well-being of households. Both channels work through the changes in the interest rate and the wage rate resulting from crowding out and distortionary taxation. First, a higher interest rate facilitates self-insurance of private households, since saving yields a higher return (Aiyagari and McGrattan, 1998). Put differently, government debt has an insurance effect because the price of the riskless production factor (capital) increases, while the price of the risky factor (labor) decreases (Davila et al., 2011; Gottardi et al., 2010). Second, government debt also affects the distribution of consumption via the composition of income, because households that receive capital income benefit and households that mainly rely on labor income lose. The insurance and the income composition channel might have counteracting effects on total welfare, since the households that profit the most from additional insurance are the consumption-poor, which also suffer the most from a decline in wages.

Our result suggests that the negative impact government debt has on welfare via efficiency losses and the income composition channel overrides the positive effect of additional insurance. Our conclusion that the government should optimally provide additional capital as a means of production instead of issuing debt stands thus in contrast to the view of Aiyagari (1995). In this seminal contribution, he emphasizes that the precautionary saving motive present in incomplete markets economies leads to an overaccumulation of capital compared to the complete market benchmark, suggesting that any government policy that reduces capital could be welfare improving. However, if one takes the fact that markets are incomplete as given, as we do here, a social planner could improve welfare by providing additional private capital. In this sense our finding is in line with the recent contributions of Davila et al. (2011) and Gottardi et al. (2010). Both papers find that competitive equilibrium allocation of resources is constrained inefficient, and that there is in fact an underaccumulation of capital. Our paper contributes to this literature by showing that a benevolent government can increase aggregate welfare by accumulating assets and buying private bonds. This policy can - at least partly - make up for the underaccumulation of assets in the private sector.

We show that our main result - namely that the government should hold assets instead of debt - hinges on the fact that wealth and income in the United States are very unequally distributed across the population. Because households that are consumption-poor also hold no wealth or are even in debt, the positive insurance effect of government debt is weak in terms of aggregate welfare. This explains why we find substantial negative welfare effects of government debt, whereas Aiyagari and McGrattan (1998) and in particular Flodén (2001) conclude that the opposing effects almost cancel out, leading to only weak overall welfare effects of government debt. Compared to these authors, we explicitely target the high wealth and earnings inequality observed in the U.S. in our calibration procedure, following Castañeda et al. (2003). Looking explicitly at different wealth-groups in the population shows an even stronger effect for those individual groups. Paying back government debt and accumulate assets benefits in particular poor agents. The reason is that they depend primarily on wage income. For this group, the positive effects of crowding in (more private capital) in form of higher wages thus outweigh the negative effects of a lower interest rate.

Another important contribution of our paper is that we are also able to show that the long-run welfare gains that can be achieved by reducing government debt with respect to the status quo can outweigh the short-run losses that occur over the transition, as long as the government uses the right set of policy instruments. We propose the policy to reduce government debt by taxing capital highly in one period, and then reduce either capital taxes or labor taxes from then onwards. This policy is inspired by Greulich and Marcet (2008), who show theoretically that in a model with wealth heterogeneity but without borrowing constraints it is Pareto-optimal to leave capital income taxes high in the short run, but reduce them to zero in the long run.

In the standard incomplete market setting, borrowing constraints are exogenous and thus invariant to public policy. However, there are good reasons to believe that financial market conditions that determine borrowing restrictions also react to policy changes. Therefore we endogenize borrowing limits by assuming that households can default on their financial liabilities. Upon default, households are excluded from future borrowing and lending forever. As in Zhang (1997), Ábrahám and Cárceles-Poveda (2011) and Ábrahám and Cárceles-Poveda (2010), borrowing constraints are set such that households are indifferent between defaulting and participating in financial markets, so that there is no default in equilibrium. If the government accumulates assets, the resulting fall in the interest rate makes default less attractive, as a lower interest rate makes it easier for households to service their debt. As a consequence, borrowing limits become laxer, and the fraction of the population that is in debt increases. We find that this effect causes the optimal long run level of government assets to be even higher compared to the case when borrowing limits are exogenous.

In line with Attanasio and Ríos-Rull (2000), Ábrahám and Cárceles-Poveda (2011), Ábrahám and Cárceles-Poveda (2010) and Krueger and Perri (2010) our results underline the importance of endogenizing the borrowing limit for analyzing the impact of public policy.

Our paper is related to the recent literature analyzing the macroeconomic consequences of the recent financial crisis through the lens of incomplete markets models. Gomes et al. (2010) focus on quantifying the cost of the increase in government debt during the episode of the 'bailout'. For their analysis, they look at incomplete market models with aggregate uncertainty. They do not perform a welfare analysis. Oh and Reis (2011) argue that the fiscal rescue package after the financial crisis mainly resulted in an increase in targeted transfers. Therefore they analyze the impact of targeted transfers on aggregate economic activity in an incomplete markets model with sticky prices. Interestingly, they also point out that in the United States between 2007 and 2009, public debt increased but private debt fell. This is consistent with our extended version of the model, where a tightening of private borrowing conditions results from higher interest rates caused by high public debt. In general however, we view our work with its focus on government debt and welfare analysis as complementary to those recent papers.

The remainder of the paper is structured as follows. We present the baseline model in the next section. In section 3, we discuss the role of borrowing constraints by showing that Ricardian equivalence still holds under incomplete markets, lump sum taxes and without borrowing constraints. In section 4 we discuss the calibration of the model. Section 5 shows the quantitative results. Section 6 concludes.

# A.2 The Baseline Model

The economy we consider is a neoclassical growth model with incomplete markets where households face uninsurable income shocks, as in Aiyagari (1994). Only one-period risk-free bonds are available for households to self-insure against income shocks. Bonds are provided by either firms, in which case they can be interpreted as claims to physical capital $k_t$, or by the government, which issues government bonds $b_t$ (as in Aiyagari and McGrattan, 1998; Flodén, 2001). As we assume no aggregate risk, claims on physical capital and government bonds are perfect substitutes and thus yield the same return $r_t$.[1] In the following, we present the household sector, the firm sector and the government sector in greater detail.

## A.2.1 Household Sector

The economy is populated by a continuum of ex-ante identical, infinitely lived households with total mass of one. Households maximize their expected utility by making a series of consumption, leisure and savings choices subject to a budget constraint and a borrowing limit on assets. In period $t = 0$, before any uncertainty has realized, their expected utility is given by

$$U(\{c_t, l_t\}_{t=1,2,\dots,}) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

where $\beta$ is the subjective discount factor. The per-period utility function, $u(.)$, is assumed to be strictly increasing, strictly concave and continuously differentiable. Additionally the first derivative is assumed to satisfy the following limiting (Inada) conditions:

$$\begin{aligned}
\lim_{c\to 0} u_c(c,l) &= \infty, \ \lim_{c\to\infty} u_c(c,l) = 0 \\
\lim_{l\to 0} u_l(c,l) &= \infty
\end{aligned}$$

[1]In Gomes et al. (2008, 2010), government bonds and private capital are imperfect substitutes due to aggregate uncertainty.

The idiosyncratic shocks to household productivity, $\epsilon_t$, follow a Markov process with transition matrix $\pi(\epsilon'|\epsilon)$.

A household thus faces the following per-period budget constraint:

$$c_t + a_{t+1} = y_t$$

where $a_{t+1} = k_{t+1} + b_{t+1}$ denotes the sum of the claims on physical capital and government bonds in $t+1$ bought by the household in period $t$ and $y_t$ is the agent's individual (after-tax) income. The households asset choice is restricted by an ad-hoc borrowing limit:

$$a_{t+1} \geq \underline{a}$$

Let $\theta(a, \epsilon)$ denote the joint distribution of asset holdings and productivity shocks. Let $\Gamma$ denote the transition function which maps the current distribution $\theta_t(.,.)$ into a new distribution $\theta_{t+1}(.,.)$:

$$\theta_{t+1}(a_{t+1}, \epsilon_{t+1}) = \Gamma[\theta_t(a_t, \epsilon_t)] \tag{A.1}$$

The government can tax labor income at some proportional tax rate, $\tau_{l,t}$, as well as financial income at some proportional tax rate, $\tau_{a,t}$, and can redistribute income via lump sum transfers, $\chi_t$. We assume that only non-negative financial income is taxed or in other words there are no proportional subsidies in the face of financial losses. More precisely, we define the tax on financial income $\tau_a$, as follows:

$$\tau_{a,t}(a_t) = \begin{cases} \bar{\tau}_{a,t} \text{ if } a_t \geq 0 \\ 0 \text{ if } a_t < 0 \end{cases}$$

The after-tax interest rate is therefore given by $\overline{r}_t = (1 - \tau_{a,t}(a_t))r_t$. The after-tax wage rate is given by $\overline{w}_t = (1 - \tau_{l,t})w_t$ where $w_t$ is the price of labor in the economy. After-tax income is thus given by:

$$y_t = \overline{w}_t \epsilon_t (1 - l_t) + (1 + \overline{r}_t) a_t + \chi_t$$

The optimization problem of the household in recursive formulation looks as follows (see the Appendix for the derivation of a detrended formulation):

$$\begin{aligned} W(a, \epsilon; \theta) &= \max_{c,l,a'} \left\{ u(c, l) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W(a', \epsilon'; \theta') \right\} \qquad \text{(A.2)} \\ \text{s.t. } c + a' &= \overline{w}\epsilon(1 - l) + (1 + \overline{r})a + \chi \\ a' &\geq \underline{a} \\ \theta' &= \Gamma[\theta] \end{aligned}$$

### A.2.2 Welfare Measure

To answer the normative question of which level of government debt would be optimal, we have to define a welfare criterion. Following the previous literature as for example, Aiyagari and McGrattan (1998) and Flodén (2001), we use as a welfare criterion the aggregate optimal value function:

$$\Omega = \int W(a, \epsilon; \theta) d\theta(a, \epsilon)$$

This criterion can either be interpreted as (1) a utilitarian social welfare function where every individual has the same weight for the planner, (2) a steady-state ex ante welfare of an average consumer before realizing income shocks or initial asset holdings or (3) the probability limit of the utility of a infinitely lived dynasty where households utilities are altruistically linked to each other (for more details see Aiyagari and McGrattan, 1998).

Because it is easier to interpret, we compute the average consumption equivalent change in welfare. A more precise definition is given in the Appendix.

### A.2.3 Production Sector

The production sector consists of a representative firm which uses capital, $K_t$, and labor, $L_t$, to produce output, $Y_t$:

$$Y_t = F(K_t, X_t L_t)$$

where $X_t$ denotes exogenous labor-augmenting technological progress.[2] This technology is assumed to grow exogenously at a constant rate $X_{t+1} = (1 + g)X_t$. For simplicity we normalize initial technology to $X_0 = 1$, such that:

$$X_t = (1 + g)^t$$

The production function $F(.,.)$ is assumed to have standard properties. The firm has to rent capital and labor from the owners at prices $w_t$ (wage = price of labor) and $r_{Kt}$ (rental rate = price of capital). Thus competitive factor markets and profit maximizing firms imply the following prices of labor and capital:

$$w_t = F_L(K_t, X_t L_t) \quad \text{(A.3)}$$

$$r_t = F_K(K_t, X_t L_t) - \delta \quad \text{(A.4)}$$

where $r_t$ is the rate of return net of depreciation. In equilibrium this rate of return has to be equal across all assets and thus determines the "interest rate" on assets.

### A.2.4 Government Sector

The government has to finance a fixed amount of government spending $G$ and the total transfers to households, $TR$ by issuing new government bonds,

[2]The presence of technological progress matters for the calibration of the discount factor. Without growth one would need a higher discount factor in the model to produce an realistic interest rate. The discount factor plays an important role for assessing the optimal value of government debt (Aiyagari and McGrattan, 1998 and Flodén, 2001).

$B_{t+1}$ and levying taxes on positive asset and labor income. Furthermore it also has to pay back bonds from the last period, $B_t$ and pay interest on them, $r_t B_t$. The government budget constraint is thus given by:

$$G + r_t B_t + TR = B_{t+1} - B_t + \tau_l w_t L_t + \bar{\tau}_a r_t \widehat{A}_t \tag{A.5}$$

where $\widehat{A}_t \geq A_t$ is the tax base for the asset income tax. As explained above taxes are only levied on positive financial income (no proportional transfers from the government for indebted people) and thus the tax base is defined as:

$$\widehat{A}_t = \int\limits_{a_t \geq 0} a_t d\theta(\epsilon_t, a_t)$$

Aggregate transfers have to equal the sum of all individual transfers:

$$\int \chi d\theta(\epsilon_t, a_t) = TR$$

### A.2.5 Recursive Competitive Equilibrium

Using the characterization of the three sectors we can now define the recursive competitive equilibrium.

**DEFINITION A.1.** *Given a transition matrix $\pi$, a certain sequence of government bond issues $\{B_t\}_{t=0}^{\infty}$, the time invariant level of government expenditures $G$, a certain sequence of capital income taxes $\{\tau_a(a_t)\}_{t=0}^{\infty}$, a certain sequence of labor income taxes $\{\tau_{l,t}\}_{t=0}^{\infty}$ and an initial distribution of the idiosyncratic productivity shocks and of the asset holdings $\theta_0(\epsilon_0, a_0)$* a recursive competitive equilibrium *is defined by a law of motion $\Gamma$, factor prices $(r_t, w_t) = (r(K_t), w(K_t))$, the value function $W = W(\theta, a, \epsilon)$ and policy functions $(c, a') = (\gamma(\theta, a, \epsilon), \zeta(\theta, a, \epsilon))$ such that*

1. *Households' utility maximization problem is defined in equation (A.2).*
2. *Competitive firm maximize profits, such that factor prices are given by (A.3) and (A.4).*

3. *The government budget constraint as defined in equation (A.5) holds.*

4. *Factor and goods markets have to clear:*

   - *Labor market clearing:*

$$N_t = \int \epsilon_t (1 - l_t) d\theta(\epsilon_t, a_t) = L_t$$

   - *Asset market clearing:*

$$A_{t+1} = \int a_{t+1} d\theta(\epsilon_t, a_t) = K_{t+1} + B_{t+1}$$

   - *Goods market clearing:*

$$\int c_t d\theta(\epsilon_t, a_t) + G + I_t = F(K_t, X_t L_t)$$

   *where investment, $I_t$ is the sum of private investment and public investment:*

$$I_t \equiv K_{t+1} - (1 - \delta)K_t + B_{t+1} - (1 + r_t)B_t$$

5. *Rational expectations of households about the law of motion of the distribution of shocks and asset holdings, $\Gamma$ reflect the true law of motion, as defined in (A.1).*

## A.3 Ricardian Equivalence and the Role of the Borrowing Limit

The assumption that households face binding borrowing constraints is central to the effect of government debt on the economy that we emphasize in this paper. It is a well known result that under complete markets and lump sum taxation government debt is neutral because agents foresee future tax changes and adapt their savings behaviour accordingly. But also under incomplete markets (and lump sum taxes) Ricardian equivalence holds, if there are no "ad hoc" borrowing limits.[3] Thus it is not the market incompleteness per se, but rather the combination of market incompleteness and borrowing constraints that leads to an effect of government debt on the economy. Thus one has to distinguish clearly between effects arising due to the presence of borrowing constraints (in other words "crowding out") and effects due to tax distortions. Heathcote (2005) makes a similar point in a related contribution.[4] The goal of this section is to clearly show this point analytically. More precisely, we will show that government debt is neutral for the case of natural borrowing limits and explicitely make clear why "ad hoc" borrowing limits lead to non-neutrality of government debt.

In the incomplete markets model described above the maximization problem of the household, if there are no "ad hoc" borrowing limits can be characterized by the following Bellman equation (in recursive notation):

$$\begin{aligned} W(a,\epsilon;\theta) = \max_{l,a',c} & \left\{ u(c,l) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W(a',\epsilon';\theta') \right\} \\ \text{s.t. } & \overline{w}\epsilon(1-l) + (1+\overline{r})a + \chi = a' + c \\ & c \geq 0 \end{aligned}$$

Suppose the government issues supplementary debt $\Delta b$ and redistributes the proceeds as a supplementary transfer, such that $\chi_{new} = \chi + \Delta b$. Furthermore suppose that the additional taxes necessary to pay the interest payments in future periods will be lump sum. It is easy to show that this policy doesn't affect the permanent income of an agent. Although he gets a transfer now,

[3] In this case there is still a so called natural borrowing limit arising from the fact that consumption must be positive (see also Aiyagari, 1994).

[4] Heathcote (2005) analyzes the quantitative short run effects of changes in the timing of proportional income taxes in heterogeneous agent economies with incomplete markets. He also distinguishes between effects via tax distortions and effects arising due to the presence of borrowing constraints.

he has to pay higher taxes in the future whose present value is equal to the transfer received. Therefore to smooth consumption the household can simply save $a'^{*}_{new} = a'^{*} + \Delta b$, where $a'^{*}$ denotes the optimal savings without the supplementary debt issuance. This will leave the household's budget in the period of the issuance of government debt unaffected as the additional debt cancels out:

$$\begin{aligned} c + a'^{*}_{new} &= \overline{w}\epsilon(1-l) + (1+\overline{r})a + \chi_{new} \\ c + a'^{*} + \Delta b &= \overline{w}\epsilon(1-l) + (1+\overline{r})a + \chi + \Delta b \end{aligned}$$

In future periods his budget is also unaffected as the additional income from savings can be used to pay the additional lump sum tax.(Note that the necessary additional lump sum tax is reduced by the higher income from asset income taxation $T'_{new} = r'\Delta b - \tau'_a r'\Delta b = \overline{r}'\Delta b$):

$$\begin{aligned} c' + a''^{*}_{new} + T'_{new} &= \overline{w}'\epsilon'(1-l') + (1+\overline{r}')a'^{*}_{new} + \chi \\ c' + (a''^{*} + \Delta b) + \overline{r}'\Delta b &= \overline{w}'\epsilon'(1-l') + (1+\overline{r}')(a'^{*} + \Delta b) + \chi \end{aligned}$$

Thus the optimal path of consumption and leisure of the household remains unaffected by the policy. The demand for assets is increased exactly by the amount of government debt issues such that firms face the same remaining demand for their assets as before. As a consequence the interest rate and wage rate in the economy will remain the same. Government debt is neutral.

**RESULT 1.** *Under incomplete markets, lump sum taxes, when households face no borrowing constraints, government debt is neutral.*

Under an "ad hoc" borrowing constraint however the households for which the constraint binds do not save optimally but rather $a' = \underline{a}$. Those households will thus not adapt their savings by adding the additional government debt $a'_{new} < \underline{a} + \Delta b$, because they can improve their consumption path (get nearer to the optimum) by consuming more in the period, when they are borrowing constrained and obtain the transfer. Intuitively, as there is the chance to obtain a higher income shock they might not be borrowing constrained anymore in the future. Thus they are more in need of funds today than tomorrow and use the transfer to "relax" their borrowing constraint. Therefore the aggregate demand for assets does not increase by as much as the new debt issues. The remaining demand for assets is lower and as a consequence capital is more expensive for firms. The interest rate in the economy will increase and the wage rate will decrease. Government debt is not neutral anymore. A more formal derivation of the results of this section is given in the Appendix.

In this section, we have shown that in our model binding borrowing constraints are a precondition for government debt to have non-trivial macroeconomic effects other than through accompanying changes in taxation. Given this important role of borrowing constraints we analyze the case where borrowing constraints are endogenous to public policy in the next section.

## A.4 An Extension: Endogenizing the Borrowing Limit

So far, we have assumed that borrowing limits are exogenous and thus invariant to public policy. This does not necessarily need to be the case. In this section, we follow Zhang (1997), Alvarez and Jermann (2000), Kehoe and Levine (2001), Krueger and Perri (2006) and Ábrahám and Cárceles-Poveda (2010) who endogenously generate borrowing limits by assuming that households cannot commit to honor their debt contracts. In a related contribution, Ábrahám and Cárceles-Poveda (2011) show that endogenizing borrowing limits matters in an environment with incomplete markets for the optimal mix of capital and labor taxes.

In line with the literature on limited commitment, we assume that if households default, they are excluded from future borrowing and lending (autarky).

The value of autarky can be expressed as follows:

$$\begin{aligned}
V(\epsilon;\theta) &= \max_{c^{aut},l^{aut}} \left\{ u(c^{aut}, l^{aut}) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) V(\epsilon';\theta') \right\} \\
\text{s.t. } c^{aut} &= (1+\lambda)\overline{w}\epsilon(1-l^{aut}) + \chi \\
\theta' &= \Gamma[\theta]
\end{aligned}$$

where $\lambda$ is a parameter describing a consumption gain (if $\lambda > 0$), which translates into a utility gain, resulting in autarky. $\lambda$ captures, in reduced form, the differences between default regulation in reality and in our model. For example, a $\lambda > 0$ could arise because in reality, exclusion from financial markets is temporary only, while exclusion is permanent in our model.[5] We calibrate the value of $\lambda$ to match the number of people in debt.[6]

For households that have not defaulted yet, the optimization problem can be stated as follows:

$$\begin{aligned}
W(a,\epsilon;\theta) &= \max_{c,l,a'} \left\{ u(c,l) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W(a',\epsilon';\theta') \right\} \qquad \text{(A.6)} \\
\text{s.t. } c + a' &= \overline{w}\epsilon(1-l) + (1+\overline{r})a + \chi \\
a' &\geq \xi(\epsilon;\theta) \text{ for all } \epsilon'|\epsilon \text{ with } \pi(\epsilon'|\epsilon) > 0 \\
\theta' &= \Gamma[\theta]
\end{aligned}$$

It is the same problem as problem A.2 above, except for the fact that the household's borrowing limit $\xi$ is now a function of the distribution of assets ($\theta$) and the realization of the income shock $\epsilon$.

[5] Assuming instead a temporary exclusion from financial markets would make the model more complicated and does not yield any qualitative value added to the analysis.

[6] There may also be additional costs related to default (e.g. social stigma) which are not modeled. This would make $\lambda$ smaller. In the calibration section, we find that for our setting to be able to match the number of people in debt we need $\lambda > 0$.

More precisely, the borrowing limit is defined as follows:

$$\begin{aligned}
\underline{a}(\epsilon;\theta) &= \{\underline{a} : W(a = \underline{a}, \epsilon;\theta) = V(a = 0, \epsilon;\theta)\} \\
\xi(\epsilon;\theta) &\equiv \sup_{\epsilon':\Pi(\epsilon'|\epsilon)>0} \{\underline{a}(\epsilon';\Gamma(\theta))\}
\end{aligned} \tag{A.7}$$

We impose that the borrowing limits in equilibrium are set such that there is no default.[7] The recursive competitive equilibrium in this extended model is thus similar to the one stated in the previous section, with an additional no-default condition.

**DEFINITION A.2.** A recursive competitive equilibrium with endogenous borrowing limits *is defined as a recursive competitive equilibrium as defined by Definition C.5, where additionally*

- *Borrowing limits are set such there is no default as given by equation (A.7).*

Ábrahám and Cárceles-Poveda (2010) show that if the period utility function is unbounded below, then equation (A.7) defines a unique and finite default threshold. They also show that the default threshold on individual capital holdings is non-positive. Intuitively, the borrowing limit, denoted by $\underline{a}$, is defined such that the value of being in autarky is just equal to the value of keeping the debt and staying in the market, if a household is actually on the borrowing limit. Or in other words the borrowing limit is the lowest possible asset holdings of an agent so that he still prefers holding on to the debt contract and staying in the market rather than not repaying the debt but being excluded. Depending on the income state, $\epsilon$, that realizes we will have a different borrowing limit.[8]

[7] The general mechanism - that borrowing conditions are tighter (looser) when the interest rate is higher (lower) - would also hold if we allowed for borrowing in equilibrium. As our aim is to model the endogenous reaction of the borrowing conditions and not the default behaviour of households we thus assume a model of limited commitment with no default in equilibrium. See Livshits et al. (2007) and Chatterjee et al. (2007) for a discussion of U.S. bankruptcy laws and a quantitative model of consumer default.

[8] Ábrahám and Cárceles-Poveda (2010) also show that the same borrowing limits would result by assuming perfectly competitive financial intermediaries. The "effective" borrow-

## A.5 Calibration

As a benchmark, we calibrate our model with endogenous labor supply to the long run average of the U.S. economy.

### A.5.1 Utility Function, Production Function and Taxes

We assume that preferences can be represented by a constant relative risk aversion utility function:

$$u(c) = \frac{(c^{\eta} l^{1-\eta})^{1-\mu}}{1-\mu}$$

We set $\mu$, the coefficient of relative risk aversion, to 2 in our benchmark calibration, which is well in the range commonly chosen in the literature (between 1 and 3).

We also assume a Cobb-Douglas production function:

$$F(K, XL) = K^{\alpha}(XL)^{1-\alpha}$$

As already mentioned above, we normalized initial technology to $X_0 = 1$, such that $X_t = (1+g)^t$. The parameter $\alpha$ of the production function is set to target a labor share of 0.7. The discount factor $\beta$ is set to target an asset-output ratio of 3.1 (cf. Cooley and Prescott, 1995 or Ábrahám and Cárceles-Poveda, 2010). The labor elasticity $\eta$ is set to target an average labor supply of 0.3. The depreciation rate is set to target an investment share of around 20 percent, which we estimate from the Penn World Tables.

ing limit that financial intermediaries will impose one period in advance, denoted by $\xi$, is chosen such that if the worst possible state (given the state today) occurs tomorrow the household will still not default. Thus they will choose the tightest of the possible borrowing limits as "effective" borrowing limit. Note that some state tomorrow could occur with zero probability given a particular state today. In this case, the associated borrowing limits can be neglected. As we will see later for our calibration, as it is possible to reach the lowest income state from any other previous state, there is thus only one relevant borrowing limit.

We take that annual depreciation rate $\delta$ to be 7 percent (see also Trabandt and Uhlig, 2009). Table A.1 shows the values of the parameters and the targets to which they are calibrated. The fiscal policy parameters are set as

**Table A.1:** Targeted Parameter Values

| Parameter | Value | Target | Data | Model |
|---|---|---|---|---|
| $\alpha$ | 0.3 | Labor share | 0.7 | 0.7 |
| $\beta$ | 0.9576 | $K/Y$ | 3.1 | 3.1003 |
| $\eta$ | 0.3092 | Labor, $L$ | 0.3 | 0.30001 |
| $\delta$ | 0.07 | $I/Y$ | 0.225 | 0.19 |
| $g$ | 0.02 | Output growth | 0.02 | 0.02 |
| $\lambda$ | 0.083 | % of HH with $a \leq 0$ | 0.24 | 0.2473 |

found by Trabandt and Uhlig (2009), which are similar to the ones found in Mendoza et al. (1994) (see Table A.2).

**Table A.2:** Parameter Values taken from the Literature

| Parameter | Value |
|---|---|
| Debt to GDP ratio, $b$ | 0.670 |
| Labor tax, $\tau^l$ | 0.28 |
| Capital tax, $\tau^k$ | 0.36 |
| Transfers, $\chi$ | 0.083 |

## A.5.2 Income Process

We follow Castañeda et al. (2003) by calibrating our model to the Lorenz curves of U.S. earnings and wealth as reported by the 2007 Survey of Consumer Finances (SCF). This is in contrast to the previous literature, where the earnings process was measured directly from the data. We choose this different procedure because it allows us to find an income process that is consistent with both the aggregate and the distributional aspects of the data on earnings and wealth (Castañeda et al., 2003). A desirable feature is that such an income process produces a sufficiently large amount of wealth accumulation by earnings-rich households to explain the high cross-sectional inequality in wealth holdings seen in the data. Since we do not model housing, we define wealth as net financial assets excluding housing and other real assets (see also Ábrahám and Cárceles-Poveda, 2010). Earnings are defined in a way to be as close as possible to the concept of earnings in the model, i.e. labor earnings such as wages and salaries plus a fraction of business income before taxes and without government transfers (for a more detailed definition see Díaz-Gímenez et al., 1997). Note that we have to specify a vector of 4 income states and a $4x4$ transition matrix, which after normalization of the vector of income states leaves us with 15 targets to specify. Notice that this is much less than the actual number of potential targets that we could pick. Instead of targeting 15 specific points, we searched for a set of parameter values such that the Lorenz curves of earnings and wealth generated by the model are similar to the ones observed in the data. As an illustration how close our calibration achieves in matching the data we report the quintiles of the wealth distribution and the earnings distribution in Table A.3.

More precisely, we find the following vector of income states:

$$s = \{0.055, 0.551, 1.195, 7.351\}$$

It should be noted that the highest income state is more than 130 times as high as the lowest income state. Furthermore, we get the following transition matrix for the income states:

$$\Pi = \begin{bmatrix} 0.940 & 0.040 & 0.020 & 0.000 \\ 0.034 & 0.816 & 0.150 & 0.000 \\ 0.001 & 0.080 & 0.908 & 0.012 \\ 0.100 & 0.015 & 0.060 & 0.825 \end{bmatrix}$$

**Table A.3:** Distributional properties at benchmark stationary economy

| | Q1 | Q2 | Q3 | Q4 | Q5 |
|---|---|---|---|---|---|
| | | Wealth (financial assets) | | | |
| Data | −1.60% | 0.10% | 1.64% | 8.29% | 91.57% |
| Benchmark Calibration | −1.57% | 0.88% | 3.92% | 7.23% | 89.54% |
| Model fitted to AR(1) | 3.24% | 10.07% | 16.96% | 25.71% | 44.03% |
| | | Earnings | | | |
| Data | −0.40% | 3.19% | 12.49% | 23.33% | 61.39% |
| Benchmark Calibration | 0.00% | 2.38% | 12.58% | 22.73% | 62.31% |
| Model fitted to AR(1) | 1.21% | 9.70% | 16.18% | 26.85% | 46.07% |

* Quintiles (Q1-Q5) denote wealth (resp. earnings) of a group in percent of total wealth (resp. earnings).

As can be seen from the transition matrix, there is a 10 percent probability of moving from the highest income state today to the lowest income state tomorrow. This generates a strong saving motive for income-rich households, leading to the high degree of wealth inequality that we also observe in the data. The same mechanism is also present in the transition matrix found by Castañeda et al. (2003).

In the rows labelled "Model fitted to AR(1)"in table A.3, we report the earnings and wealth inequality that would result if the earnings states and the transition matrix where fitted to replicate an AR(1) process with persistence $\rho = 0.6$ and variance $\sigma = 0.3$ (the values used by Aiyagari and McGrattan (1998) in their benchmark specification). It is important to notice that this method does not generate the degree of wealth and earnings inequality that we observe in the data.

### A.5.3 Borrowing Limits

We calibrate the ad-hoc borrowing limit to match the percentage of households with negative or zero financial assets in the 2007 SCF (24 percent). We find a borrowing limit of $\underline{a} = -0.3$.

In the extension of the model with limited commitment we can generate this borrowing limit endogenously by setting the parameter defining utility in the autarky state to $\lambda = 0.083$.

Recall that $\lambda > 0$ can be interpreted as a relaxation of the autarky state, which suggests that our modelling of autarky is "too harsh" with respect to the data.

## A.6 Results

We are now ready to compute the optimal amount of government debt with the help of our quantitative model. In the first section, we focus on the long run consequences of government debt on welfare. That is, we compare the aggregate welfare of stationary equilibria that are associated with different levels of government debt, ignoring the welfare effects that arise along the transition between different stationary equilibria. This allows us to compare our results with previous contributions that analyzed the optimal level of government debt by comparing stationary equilibria as well (see e.g. Aiyagari and McGrattan, 1998 and Flodén, 2001). Second, we believe that this is a useful excercise to gain intuition about how government debt can affect welfare in general. In section A.6.3, we also include the transition path into our welfare calculations.

### A.6.1 Long Run Average Welfare

When markets are incomplete, an increase in government debt has positive and negative effects on aggregate welfare. In this section, we use our calibrated model to compute the net effect of these counteracting effects.

Remember that due to the presence of borrowing constrained agents savings will not be increased by the amount of additional government debt, but less, such that the bond price decreases or equivalently the interest increases. This implies crowding out of private capital and thus as firms produce less and need less labor as a consequence, a lower wage rate. The reverse happens when government debt is reduced (lower interest rate, higher wage rate). Additionally long run taxes change and affect welfare. For convenience, we now summarize the three main welfare effects of government debt in our framework. Consider for simplicity an increase in government debt (generally the reverse will hold true for a decrease):[9]

1. Level effect: As discussed above government debt leads to crowding out of private capital. If the capital stock in the benchmark economy is too low with respect to its efficient level, this crowding out leads to lower welfare. The behaviour of tax rates is non-monotonic, as will be discussed below. The level welfare effect of government debt through taxes depends if disposable income is higher or lower as a consequence.

2. Insurance effect: If the interest rate increases and the wage rate falls, the uncertain component of income, namely labor income, is reduced relative to the certain component (capital income). This leads to less uncertainty of consumption and thus to welfare gains for households from an ex ante perspective.

3. Income composition effect: As the interest rate rises relative to the wage rate, wealth-rich households gain compared to wealth-poor households, because the latter depend more on labor income. Because the wealth-poor are also the consumption-poor, this leads to an aggregate welfare loss since marginal utility is higher for households with low levels of consumption.

[9] Our labelling of the different effects follows Flodén (2001). He distinguishes between a "level" effect, an "uncertainty" effect and an "inequality" effect. We chose to label the redistributive effect "income composition effect", because we would like to capture how the redistribution arises, namely because of the different compositions of income between households in this model.

Figure A.1 shows the average consumption equivalent change in welfare over the different stationary equilibria. Figure A.1 reveals that it is optimal for

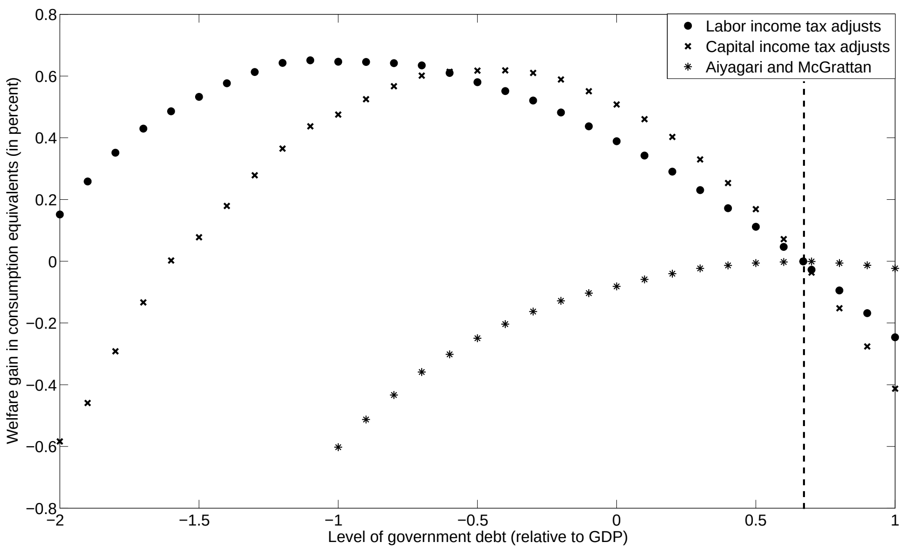


**Figure A.1:** Consumption Equivalent Change in Welfare With Respect to Benchmark

the government to hold assets, either of a value of around 50 percent of GDP, if debt is financed by changes in capital income taxation, or of a value of around 110 percent of GDP, if debt is financed by changes in labor income taxation. This result is in contrast to the previous literature, notably Aiyagari and McGrattan (1998), who find that the government should optimally issue debt of around 60 percent of GDP.

The reason for the different outcomes lies in the different calibration of the income process. Recall from table (A.3) that our calibration implies that the degree of wealth inequality generated by our model is close to the one that is observable in the U.S., where a large fraction of the population holds almost no assets at all. This implies that these households will not benefit from an increase in the interest rate associated with an increase in debt. Consequently, we find that the level effect and the income composition effect of an increase in government debt outweigh the insurance effect.

This finding is line with the recent results in Davila et al. (2011), who compute the constrained efficient level of capital in a model with incomplete markets. As we do, they conclude that it is the income composition of the consumption-poor households that matters. If those households depend mainly on labor income which is subject to idiosyncratic productivity shocks, the constrained efficient allocation involves a larger stock of capital than the market economy delivers. The income composition effect is then more decisive for aggregate welfare than the insurance effect. Figure A.1 also reveals

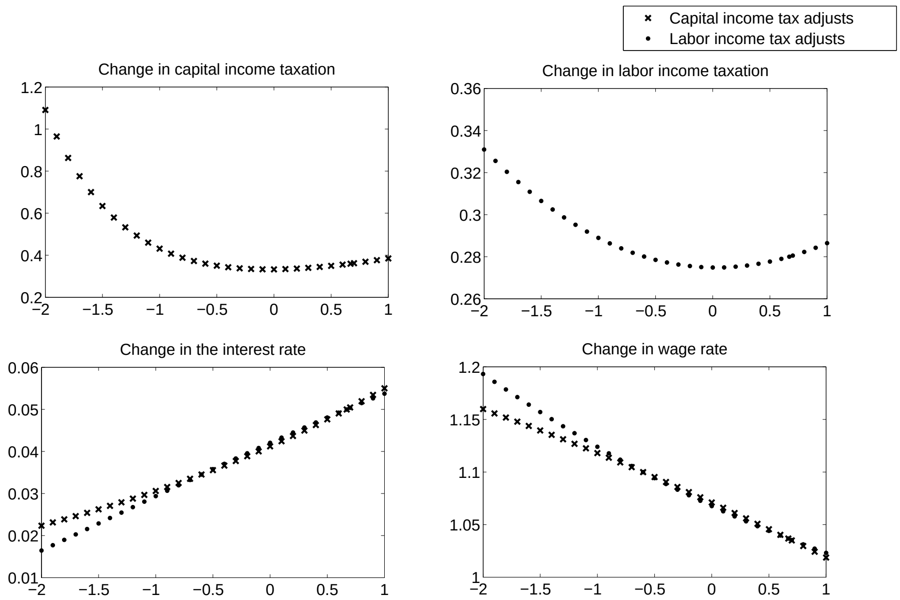


**Figure A.2:** Implied Tax Rates, Interest Rates and Wage Rates for Stationary Equilibria With Different Debt Levels

that depending on whether debt is financed by labor taxes or capital taxes, we get a different optimal level of government assets. In order to understand the differences, we first need to understand the general behaviour of tax rates after a change in government debt. First, recall that from the government budget constraint (A.5), an increase in government debt implies that the government needs to levy more taxes in order to pay the additional interest due, if the government expenditures and transfers are kept fixed. When government debt is reduced however the behaviour of taxes is non-monotonic, decreasing for modest reductions, but increasing again for higher reductions. The reason is that a lower interest rate, as a consequence of lower government debt reduces the incentive to save. Therefore assets of the households and consequently also the tax base for the capital income tax of the government.

Now, back to the question why the optimal public asset levels are different depending on the form of taxation that is used to equalize the budget. Figure A.2 shows that the capital income tax has to change more dramatically to keep the budget balanced after a change of government debt, compared to a situation when the labor income tax adjusts. Consider the role of capital taxation when the government increases its assets (i.e. reduces government debt). As we have argued above the lowering of the interest rate leads to an erosion of the tax base. This forces the government to increase the capital tax rate in order to keep its budget balanced, therefore reducing after-tax returns on capital even further. A vicious circle starts.

Now consider the role of labor taxation when the government increases its assets. Again, the interest rate falls and the wage rate rises. The combination of the two causes households to consume more leisure and to work less. However, differently from the case of capital taxation, there is no vicious circle as for capital taxation, but rather a dampening effect: the rising wage dampens the negative incentive effect of an increase of the tax rate. However, increasing the labor income tax rate also dampens the positive effect of higher wages on welfare. Clearly, after-tax wages rise less for a given reduction in government debt than in the case where capital income taxation is used. As a consequence, the optimal level of government debt is about twice as low as in the case where the capital income tax adjusts (-110 percent versus $-50$ percent of GDP).

Moving towards the optimal level of government assets leads however to relatively small aggregate welfare gains considering the effort it probably means in practice to reduce government debt. Starting from the benchmark reducing government debt relative to GDP by 10 percent (from $\frac{2}{3}$ to 0.6) increases welfare to a level equivalent to an average increase in consumption by 0.0715 percent if the capital income tax adjusts and by 0.0474 percent if the labor income tax adjusts. On overall welfare gains up to an equivalent average rise of consumption of 0.618 percent are possible for the case of capital income taxation and of 0.654 percent for the case of labor income taxation. Although the overall effect is already approximately 7.5 times larger than for the specification of Aiyagari and McGrattan (1998), it is still relatively small.

In the next section we show that those small *aggregate* welfare effects can be decomposed into substantial *group-specific* welfare effects depending on the wealth group of households that is considered.

### A.6.2 Welfare of Different Wealth-Groups: Poor, Middle Class, Rich

The policy of changing the level of government debt has different implications depending on which wealth-group is considered. Thus, to fully understand compositional effects on welfare it is useful to consider the evolution of the relative importance of wealth groups and the respective welfare changes for those groups. For this purpose let us define three types of agents according to their wealth:

1. Poor: Agents who are indebted or hold zero assets.
2. Rich: Agents who are part of the group of richest persons who together as a group hold 70 percent of total assets.
3. Middle class: Agents who are neither rich nor poor.

It is important to stress that those groups are exclusive for all specifications considered here. By definition poor households do not have any asset income and therefore care only about their labor income. The poor are thus only affected by changes in wages or labor income taxation. The rich have only little labor income compared to asset income. Therefore they care primarily about changes in the interest rate and capital income taxation. Finally, the middle class is affected by both changes in labor income and changes in capital income.

Figure A.3 shows the change in welfare at different levels of government debt for each wealth-group and the change of the relative size of each wealth-group. The first thing to note is that there are large group-specific changes

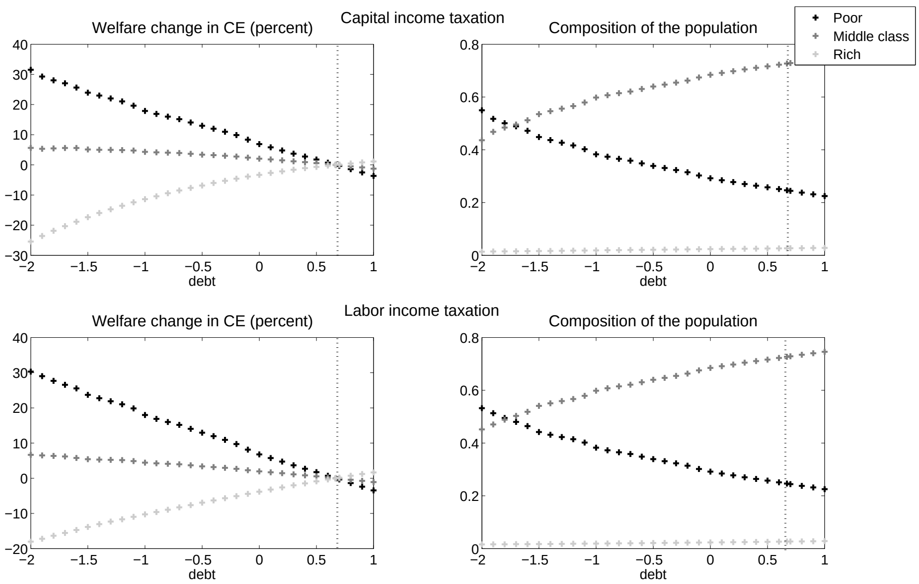


**Figure A.3:** The Compositional Effects: Welfare Changes of the Poor, the Middle Class and the Rich in Comparison

in average (group-) welfare (see the left hand side of Figure A.3). On aggregate those effects seem to average out, which explains small overall welfare effects. Secondly, a reduction of government debt is good for the poor. The average poor household can gain welfare worth a 0.735 percent (0.713 percent) rise in consumption from a 10 percent reduction in government debt, if financed by capital income taxation (labor income taxation). On overall, the average poor household can even gain welfare worth a 12.0 percent (16.9 percent) rise in consumption, if moving to the optimal debt level. Not surprisingly for the poor it would be optimal to reduce debt as much as possible by changing capital income taxation.

Thirdly, the welfare changes seem to be concave for the rich in the case of capital taxation. In other words the average rich household has much to lose if government debt is reduced, but not a lot to gain, if it is increased. The reason is that the change in taxes is non-monotonic, as mentioned in the last section. When government debt rises the rich gain from the rise in interest, but lose from higher taxes. When government debt is reduced from a certain level onwards they lose from both higher taxes and lower interest rates.

Finally, there is an effect of government debt on the relative size of wealth groups. A reduction of government debt increases the number of wealth-poor in the population relative to the middle class (see the right hand side of Figure A.3). Intuitively saving is discouraged because of lower interest rates. This increase in the number of poor leads to a falling tax base and eventually higher taxes. Thus an 'optimum' level arises from which onwards high tax distortions become overwhelming.[10]

In the next section we analyze how the results change when we also include the transitional periods into the welfare analysis.

[10]Reducing government debt thus reduces inequality in the long run, a result that was already emphasized by Flodén (2001).

### A.6.3 Welfare Over the Transition Path

In the previous section, we computed the optimal long-run level of government debt by comparing stationary equilibria. Our results suggested that the government should optimally hold assets.

One might wonder whether this conclusion still holds when we take the transition between two stationary equilibria into account. Clearly, if government debt is reduced and if the government keeps its expenditures fixed, it follows from the budget constraint that taxes have to be increased during the period during which debt is reduced.

In this section we argue that it is possible to gain from a reduction in government debt although either consumption or leisure (or both) need to be sacrificed in the short run. We assume that in period $t$, the government (unexpectedly) increases the capital income tax for one period to 100%. This upper bound is chosen such that capital owners are not expropriated. The revenues of this tax increase are then used to reduce government debt. This in turn allows the government to reduce either capital income taxes (scenario 1) or labor income taxes (scenario 2) from period $t = 2$ onwards. The experiment is inspired by Greulich and Marcet (2008), who show that in order to achieve a Pareto-optimal tax reform the government should leave capital taxes high in the short run and reduce then to zero in the long run.

We find that it is possible to reduce government debt to 56.54% in scenario 1 and to 56.47% in scenario 2 (see Table A.4). Under both scenarios, total welfare gains from reducing government debt are positive, namely +0.007% in scenario 1 or +0.026% in scenario 2. This holds despite the fact that substantial welfare costs occur during the transition, a finding that we explain in more detail in the following. Notice that it seems more promising to reduce long run labor income taxes. In order to better understand the intuition behind our results, we plotted the path of the capital stock, of labor supply,

**Table A.4:** Welfare changes over the transition path

| Kind of tax rate | New debt to GDP level | Welfare change excl. trans. | incl. trans. |
|---|---|---|---|
| Capital income tax | 56.54% | +0.106% | +0.007% |
| Labor income tax | 56.47% | +0.107% | +0.026% |

of the after-tax interest rate and of the after-tax wage rate in Figure A.4. [11] Recall from the long-run analysis above that when government debt is reduced, this crowds in private capital. This stems from the fact that borrowing constraints are binding for some households. These households do not dissave, despite the fact that taxes are expected to be lower in the future. As a result, the private capital stock rises and the equilibrium interest rate falls (see first picture of Figure A.4).

However in the short run there is an additional effect from the one-period tax increase, stemming from the fact that the capital income of one period is taxed away. This is a negative income effect, and because leisure is a normal good, households consume less leisure and increase their labor supply (see second picture of Figure A.4). This in turn leads to a short run increase in the interest rate and decrease in the wage rate (see third and fourth picture of Figure A.4). This hurts in particular borrowing constrained households, because they depend solely on their wage income. Moreover they already work the maximum hours, so they cannot respond to lower wages by working more. Because the borrowing constrained household are the consumption-poor, there are high welfare costs are associated along the transition path. Interestingly, in scenario 2 this adverse effect is dampened, because of an intertemporal substitution effect resulting from the fact that labor taxes will be lowered from period 2 onwards. Households are willing to increase leisure

[11] Note that the figure omits the first period for the after-tax interest rate, because in this period capital taxes are 100% by definition of our experiment. This means that the after-tax interest rate is 0 in the first period. This hinders the readability of the figure, so we left it out.

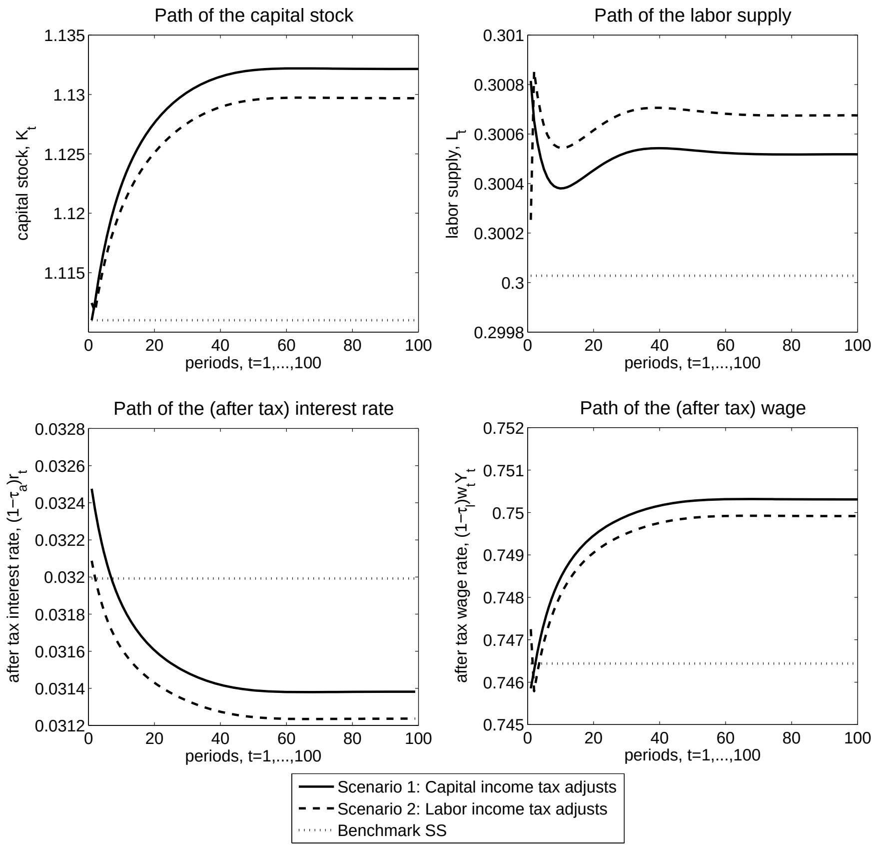


**Figure A.4:** Implied Capital Stock, Labor Supply, Interest Rates and Wage Rates for the Transition Experiment

today in exchange for less leisure tomorrow. This explains why wages are below the benchmark level for a shorter time than for scenario 1. Welfare losses are lower.

The best policy given the possibilities analyzed here is thus to reduce government debt by setting a high capital income tax today and use the lighter budget to lower labor income taxes from period 2 onwards.

However, a caveat to this analysis is that we do not explicitly aim at calculating the optimal path of capital and labor income taxes over the transition. It is possible that there is a different timing of taxes which produces even higher welfare gains. [12]

The next section shows the welfare effects of this policy for different wealth groups over the transition path.

### A.6.4 Welfare Effect on Different Wealth Groups Over the Transition Path

In this section we show that households with little wealth have the highest welfare loss during the transition, while relatively affluent households gain relative to the benchmark stationary equilibrium (see Table A.5). Interestingly, the group that benefits the most from reducing government debt in the long-run suffers the most in the short-run (and vice versa).

The intuition behind this striking result is as follows. Recall from Figure A.4 that output and thus also wages are higher in the long-run, compared to periods after the lower debt level was implemented. This is due to the fact that reducing government debt crowds in private capital. Since households are rational, they foresee this. As a result, households would like to reduce their savings today, or borrow more, respectively. Those who are borrowing constrained cannot do so, however. Thats why it is the wealth-poor who loose during the transition. Why is the wage lower in the first few periods?

[12] We have shown that the welfare effects of government debt change dramatically with a careful modelling of wealth inequality. The downside of this greater precision of the wealth distribution is however that the computation of the stationary wealth distribution is more time consuming (see for example Heer and Maussner, 2009 as a reference on different methods for computing the stationary wealth distribution and their performance in terms of precision and computing time). Therefore finding the 'optimal' path of transition which involves iterating over the whole path of interest rates, wage rates and tax rates is a difficult task. (Note that with endogenous labor there is an additional adjustment). Greulich and Marcet (2008) go in this direction by analyzing optimal capital, labor income taxes and resulting government debt holdings in a model with heterogenous agents. However they miss important aspects because they do not include individual risk and borrowing constraints. Extending their results to an Aiyagari-framework as presented here would thus be very valuable but exceeds the aim of this paper. Therefore we leave it to future research for now.

**Table A.5:** Welfare effect on different wealth groups

| Quintiles | Welfare change relative to benchmark excl. transition path | incl. whole transition path |
|---|---|---|
| Capital income tax | | |
| Q1 (20% poorest) | +0.496% | −31.067% |
| Q2 (20% – 40%) | +0.300% | −3.656% |
| Q3 (40% – 60%) | +0.182% | +6.825% |
| Q4 (60% – 80%) | +0.112% | +13.118% |
| Q5 (20% richest) | −0.798% | +49.566% |
| Labor income tax | | |
| Q1 (20% poorest) | +0.505% | −31.115% |
| Q2 (20% – 40%) | +0.310% | −3.691% |
| Q3 (40% – 60%) | +0.188% | +6.939% |
| Q4 (60% – 80%) | +0.114% | +13.232% |
| Q5 (20% richest) | −0.797% | +49.497% |

* Welfare gains in consumption units in percent.

The reason is that the funds for paying back government debt are taken from households that own wealth and partly depend on their asset income. They have to forego asset income for one period. There are two ways of dealing with such a situation, where there is less asset income: dissave or work more. In fact wealthy households will do both. Thus the general level of labor supply rises as can be seen in Figure A.4. As discussed above this is harmful for borrowing constrained agents who depend solely on labor income and virtually already work as much as possible. Notice that the loss is more pronounced for households that are hit by a particularly good or a particularly bad income shock (wealth-poor only) in the period when the reduction of the government debt takes place. This is show in Table 6 where we decompose welfare changes in the first period of the transition with respect to wealth groups and income states. Clearly, the effect of borrowing constraints is exacerbated for the wealth-poor if they are hit by a bad income shock, which explains why they loss is more pronounced for this group. However, households with the best earnings draws loose by even more, irrespectively of their wealth holdings. The reason is as follows: households with

**Table A.6:** Welfare effect in the first transition period for wealth-income-groups

| Capital income tax | | | | |
|---|---|---|---|---|
| | State 1 | State 2 | State 3 | State 4 |
| Poor | −58.143% | −46.496% | −40.456% | −75.035% |
| Middle Class | −18.012% | −19.052% | −28.656% | −73.562% |
| Rich | +12.057% | +17.777% | +6.946% | −55.212% |
| **Labor income tax** | | | | |
| | State 1 | State 2 | State 3 | State 4 |
| Poor | −58.122% | −46.435% | −40.237% | −74.948% |
| Middle Class | −18.028% | −28.351% | −28.351% | −73.470% |
| Rich | +12.135% | +17.930% | +7.225% | −55.100% |

* Welfare gains in consumption units in percent.

the highest earnings shocks expect a lower earnings in the future. This is true even though average earnings are increasing over the transition path.[13] Households in this group therefore do not want to dissave, but rather engage in precautionary saving against the event of receiving lower earnings draws in the future. This can be done both through adjusting labor supply and working longer hours and through accumulating additional savings. Because the wage rate is lower initially and households foresee a falling interest rate, they need to save more now relative to the benchmark to provide for possibly lower labor productivity in the future.

The fact that borrowing constraints play a crucial role in explaining the welfare cost of the transition suggests that the policy that directly targets transfers to the wealth-poor could reduce the welfare losses significantly, making it even more profitable to reduce government debt. We leave this for future research and conclude this section by stressing that in order to fully assess the welfare costs of fiscal policy in the short and in the long-run, understanding the interaction between borrowing constraints and wealth inequality are

[13]Recall that our calibration implies substantial difference between the highest and the other realization of the income shock.

key.

In the next section, we thus go one step further and relax the assumption that borrowing constraints are exogenous. As we will see, this gives us even higher welfare gains in the long-run.

### A.6.5 Endogenous Borrowing Limit

In section A.4 we have shown how our model can be extended to include endogenous borrowing limits by assuming limited commitment of households to repay loans. This is important because borrowing limits then have to change endogenously as a response to a change in government debt.

In this section we will thus analyze the version of our model with endogenous borrowing limits to see how this endogeneity potentially changes the results about the optimal long run level of government debt. We focus again on stationary equilibria to gain an intuition about the long run effects and be comparable to the previous literature which mainly focussed on stationary equilibria as well.

Figure A.5 shows that, in fact, there is an endogenous reaction of borrowing limits to differences in government debt. Borrowing limits are tighter in stationary equilibria where government debt is higher and looser in stationary equilibria with lower government debt. More precisely, an increase (decrease) of government debt and the subsequent rise (fall) of interest rates due to crowding out (crowding in) leads to the borrowing limit *of the lowest income group* being tighter (looser).[14] What is the intuition behind this tightening (loosening)? When there is more government debt a higher interest rate and a lower wage rate arise in the new equilibrium. How the borrowing limits react depends on the relative attractiveness of autarky. A

[14]Remember that due to our specification of the transition matrix of income shocks the borrowing limit of the lowest income group is the relevant one for any household.

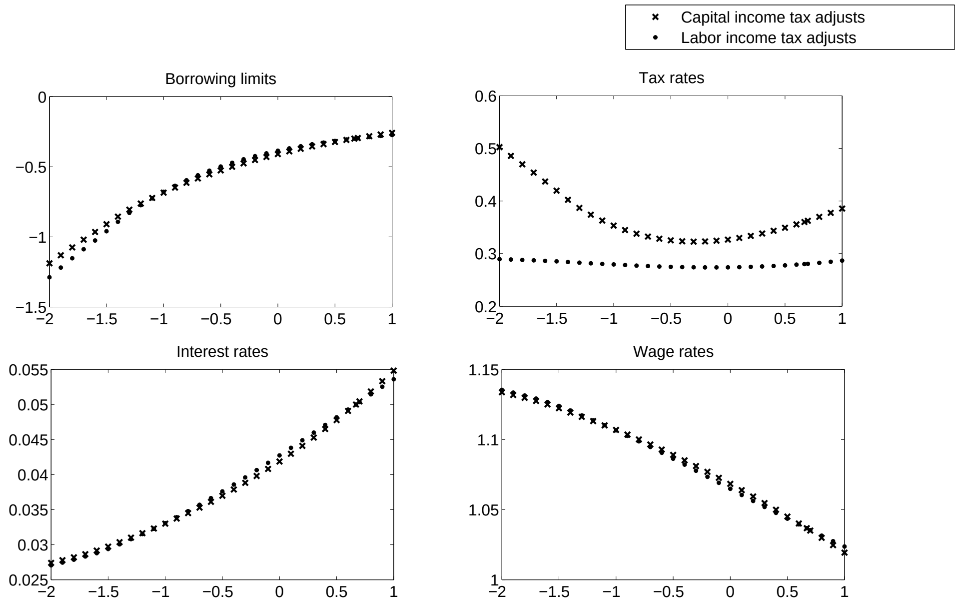


**Figure A.5:** Implied Tax Rates, Interest Rates and Wage Rates for Stationary Equilibria With Different Debt Levels in the Case of Endogenous Borrowing Limits

higher interest rate makes autarky more attractive relatively to being at the borrowing limit, because a household that is at the borrowing limit has to pay a higher debt service. The effect of the wage rate depends on how much households adjust their labor supply to smooth consumption and leisure. Households tend to use more extensively adjustments through labor supply when being in autarky than when they are part of financial markets. The reason is that in the latter case they have an additional means of smoothing consumption and leisure through savings and dissavings. A lower wage rate thus makes financial autarky less attractive, because here the household has to rely solely on adjustments to labor supply to smooth consumption and leisure. For low income households this wage effect is not important as they have to work hard regardless of being in autarky or being part of financial markets. Thus the interest rate effect dominates the wage rate effect for the lowest income group. Therefore with higher government debt and a higher interest rate the borrowing limit of this income group will become tighter and vice versa for a lower government debt.

The endogenous reaction of borrowing limits leads to a dampening effect on aggregate variables such as the interest rate, the wage rate and the tax rate (see Figure A.5). The distribution of wealth however changes faster than with exogenous borrowing limits (see Figure A.6). To gain an intuition, consider for example a reduction in government debt. If borrowing limits become looser instead of staying fixed, agents that in the case of fixed borrowing limits would like but are not able to, now can borrow more. The wealth distribution thus changes more dramatically. However this means also less a dampening effect on the changes of aggregate variables such as the interest rate and the wage rate. The reason is that the demand for savings is reduced and absorbs some of the reduction in asset supply that occurs because government debt is reduced. Remarkably the endogeneity of borrowing limits

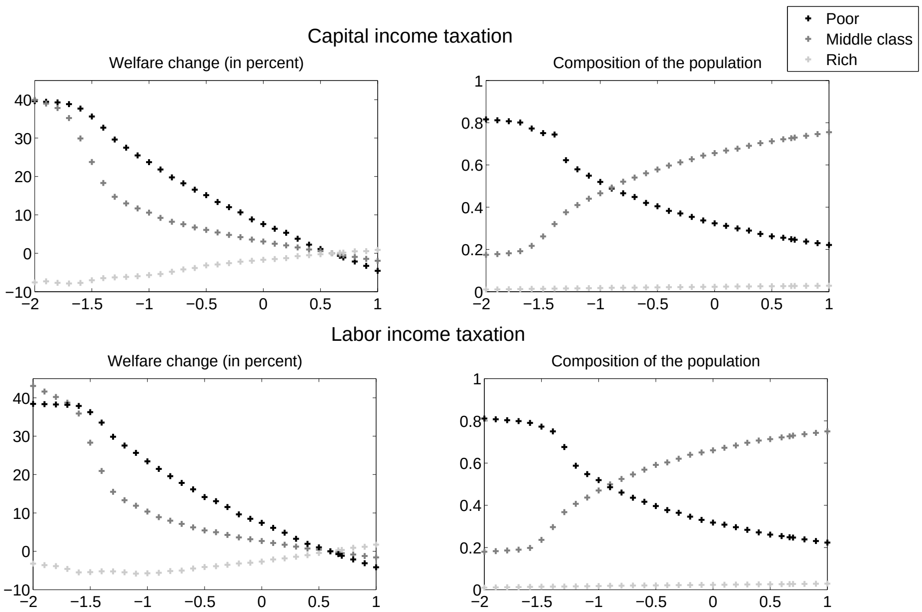


**Figure A.6:** The Compositional Effects: Welfare Changes of the Poor, the Middle Class and the Rich in Comparison in the Case of Endogenous Borrowing Limits

leads to quite different welfare effects (see Figure A.7). Firstly, the optimal

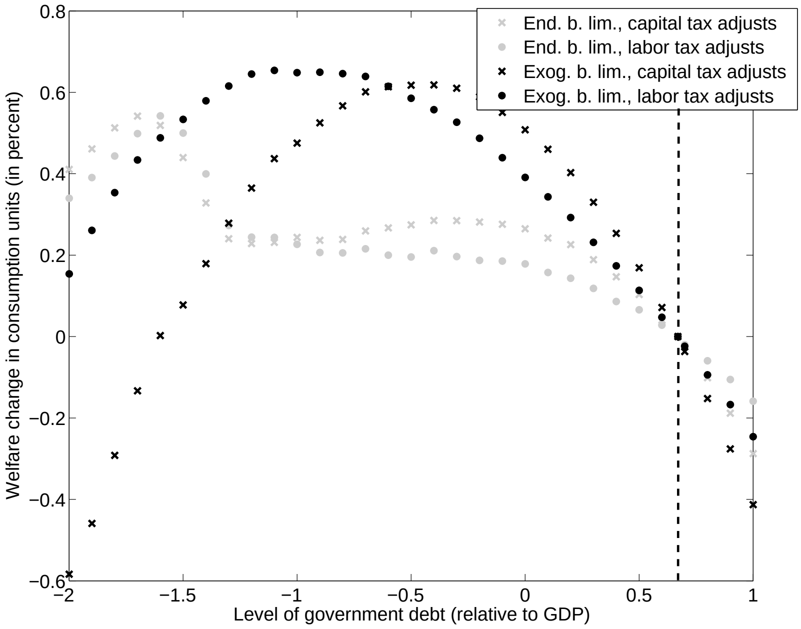


**Figure A.7:** Consumption Equivalent Change in Welfare With Respect to Benchmark in the Case of Endogenous Borrowing Limits

level of welfare is farther to the left in both the case of labor and the case of capital income taxation. It is now optimal for the government to hold assets of around $160 - 170$ percent of GDP. Intuitively, the reaction of the borrowing limits procures a higher welfare to poor agents without having to incur high tax distortions. The reason is that the reaction of the tax rate is "slower" due to the dampening effect of endogenous borrowing limits as discussed above.

Second there are sort of "double peaks" in the shape of the aggregate welfare change. To understand the reason for this difference to the case of exogenous borrowing limits, it is useful to look once more at the compositional effects. Figure A.6 shows how the welfare of different wealth-groups in the population changes. At first when there are still a minority of poor in the population the positive welfare effect of a higher wage on that group is not so strong given that the wage also changes slower. Thus at first welfare increases less steeply than in the case of exogenous borrowing limits (see Figure A.7). But subsequently as the number of poor increases faster for higher levels of government assets welfare begins to increase more steeply. Similarly to the case of exogenous borrowing limits at very high asset levels the negative effect of having a higher number of poor with generally lower welfare begins to matter more than that their welfare rises a little bit. Consequently as seen in Figure A.7 welfare starts to fall again. This mechanism translates into a different shape of the welfare change.

## A.7 Conclusion and Further Research

As shown by Davila et al. (2011) the competitive equilibrium in an incomplete markets model is 'constrained inefficient' and there is scope for public policy. In this paper we show that government debt can be used to implement an allocation nearer to the constrained efficient one. However, different than previously argued we show that if calibrating the model to the actual wealth distribution, the government should, in the long run, rather accumulate assets suggesting that the capital stock is too low under laissez-faire.

Asset-poor agents profit most from the reduction of government debt in the long run. The reason is that the increase in the wage affects them strongly as their income consists primarily of wage income. However lower interest rates mean that the incentives to save are decreased so that the number of asset-poor agents in the population increases. This leads to a falling tax base and higher taxes. An 'optimum' exists from which onwards high tax distortions become overwhelming.

To reach this new stationary equilibrium we propose a policy of setting the capital income tax to the maximum for one period, reduce government debt and using the lighter budget to reduce labor taxes afterwards. This analysis shows that at least some welfare gains can be achieved by reducing government debt. However the results about relative welfare changes are reversed relative to the long run. Asset-poor agents lose consumption and/or leisure over the transition because of temporarily lower wages and less insurance opportunities. Further research has to show if those negative transitional effects on the weakest chains of society could be alleviated by using targeted transfers.

Endogenizing borrowing limits leads to even higher optimal levels of government assets in the long run as part of the effect on the equilibrium in the asset market (and thus on the equilibrium interest rate) is absorbed by the reaction of the borrowing limits. On the one hand this makes the insurance effect weaker, on the other hand there is an additional insurance effect from the reaction of borrowing limits which permits households to borrow more. In this sense public and private insurance concur with each other. As a result government debt is even less appropriate as an insurance channel. Further research in this direction could include the analysis of transitional welfare effects or the use of transfers as insurance.

Summarizing we have shown that in an incomplete markets model which maps the high wealth inequality in the U.S. the government can achieve welfare gains by reducing government debt especially in the long run. The only source of uncertainty in our model are idiosyncratic income shocks. A possible extension would be to additionally include aggregate shocks to include a motive for tax smoothing like in Barro (1979), Lucas and Stokey (1983), Aiyagari et al. (2002) or Heathcote (2005). Another possible extension concerns the transition to the new stationary equilibrium. With respect to transitional effects it would be interesting to analyze the optimal tax mix as well as the timing of taxes for our calibration (extending results of Greulich and Marcet, 2008; Heathcote, 2005).

## A.8 Appendix to the Paper

### A.8.1 Detrended Formulation of the Household Maximization Problem

In our model, there is a balanced growth path along which variables will be growing at the rate of technology growth. To find the stationary equilibrium of the model or to compute the transition from one stationary equilibrium to another it is useful to first detrend variables with respect to this exogenous productivity growth component to obtain a formulation where variables are constant in the balanced growth equilibrium. (This procedure was also used in the earlier literature, for example by Aiyagari and McGrattan, 1998; Flodén, 2001). Denote a detrended variable by "tilde": $\tilde{x} = \frac{x}{Y}$. The present value of lifetime utility (for a Cobb-Douglas can then be denoted as follows:

$$U(\{\tilde{c}_t\}_{t=1,2,...}, \{l_t\}_{t=1,2...}) = E_0 \sum_{t=0}^{\infty} \beta^t Y_t^{\eta(1-\mu)} u(\tilde{c}_t, l_t)$$

Now using the fact that $Y_t = Y_0(1+g)^t$, where $Y_0$ is output in period 0, we can write:

$$\begin{aligned} U(\{\tilde{c}_t\}_{t=1,2,...}, \{l_t\}_{t=1,2...}) &= Y_0^{\eta(1-\mu)} E_0 \sum_{t=0}^{\infty} \beta^t (1+g)^{t\eta(1-\mu)} u(\tilde{c}_t, l_t) \\ &= Y_0^{\eta(1-\mu)} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t u(\tilde{c}_t, l_t) \end{aligned}$$

where $\tilde{\beta} = \beta \cdot (1+g)^{\eta(1-\mu)}$.

Similarly, we can find a detrended version of the household budget constraint by dividing it by $Y_t$:

$$\begin{aligned} \frac{c_t}{Y_t} + \frac{Y_{t+1}}{Y_t}\frac{a_{t+1}}{Y_{t+1}} &= \frac{\bar{w}_t}{Y_t}\epsilon_t(1-l_t) + (1+\bar{r}_t)\frac{a_t}{Y_t} + \chi \\ \tilde{c}_t + (1+g)\tilde{a}_{t+1} &= \tilde{\bar{w}}_t\epsilon(1-l_t) + (1+\bar{r}_t)\tilde{a}_t + \tilde{\chi}_t \end{aligned}$$

Also the borrowing constraint can be detrended:

$$\tilde{a}_{t+1} \geq \underline{\tilde{a}}_t$$

The resulting recursive formulation in detrended variables is given by:

$$\begin{aligned}
W(\tilde{a}, \epsilon; \theta) &= \max_{\tilde{a}', \tilde{c}, l} Y_0^{\eta(1-\mu)} u(\tilde{c}, l) + \tilde{\beta} \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W(\tilde{a}', \epsilon'; \theta') \\
\text{s.t.}\tilde{c} + (1+g)\tilde{a}' &= \tilde{\bar{w}}\epsilon(1-l) + (1+\bar{r})\tilde{a} + \tilde{\chi} \\
\tilde{a}' &\geq \underline{\tilde{a}} \\
\theta' &= \Gamma[\theta]
\end{aligned}$$

## A.8.2 Ricardian Equivalence Under a Natural Borrowing Limit but not Under a Binding "ad hoc" Borrowing Limit

The optimal sequences for consumption, leisure and savings have to satisfy the following first order conditions in every period $t$ for every possible state $\epsilon_t$:

$$\begin{aligned}
\frac{u_l(c_t^*, l_t^*)}{u_c(c_t^*, l_t^*)} &= \epsilon \overline{w}_t \\
\frac{\beta \sum_{\epsilon_{t+1}} \pi(\epsilon_{t+1}|\epsilon_t) u_c(c_{t+1}^*, l_{t+1}^*)}{u_c(c_t^*, l_t^*)} &= \frac{1}{1+\overline{r}_{t+1}} \\
c_t^* + a_{t+1}^* &= \overline{w}_t \epsilon_t (1 - l_t^*) + (1+\overline{r}_t) a_t + \chi
\end{aligned}$$

Now suppose we have a new economy with additional government debt issues $\Delta b$, a higher lump sum transfer in period 0 of $\chi_{0,new} = \chi + \Delta b$ and an additional lump sum tax $T_{t,new} = r_t \Delta b - \tau_a \Delta b = \overline{r}_t \Delta b$ for $t > 0$.

Suppose each household saves exactly $\Delta b$ in addition: $a_{new,t+1}^* = a_{t+1}^* + \Delta b$, but chooses the same consumption and leisure path given the shocks and initial asset level. Clearly this satisfies the first two conditions above. It also satisfies the budget constraint of the household as we will show now. In period 0:

$$\begin{aligned} c_0^* + a_{new,1}^* &= \overline{w}_0\epsilon_0(1-l_0^*) + (1+\overline{r}_0)a_0 + \chi_{0,new} \\ c_0^* + a_1^* + \Delta b &= \overline{w}_0\epsilon_0(1-l_0^*) + (1+\overline{r}_0)a_0 + \chi + \Delta b \end{aligned}$$

In periods $t > 0$:

$$\begin{aligned} c_t^* + a_{new,t+1}^* + T_{t,new} &= \overline{w}_t\epsilon_t(1-l_t^*) + (1+\overline{r}_t)a_{new,t} + \chi \\ c_t^* + a_{t+1}^* + (1+\overline{r}_t)\Delta b &= \overline{w}_t\epsilon_t(1-l_t^*) + (1+\overline{r}_t)(a_t + \Delta b) + \chi \end{aligned}$$

As the households choose the same consumption and leisure the wage rate will be the same as in the old economy without government debt. As the savings of each individual household are higher by $\Delta b$ aggregate asset demand has to be higher by $\Delta b$ (households being a mass of one). Supply of assets is also higher by $\Delta b$ (closed economy) such that the interest rate in the economy will be the same. If we are in a stationary equilibrium in the initial economy, we will be in the same stationary equilibrium in the economy with government debt provided that it is financed with lump sum taxes and redistributed via lump sum transfers as shown above.

Finally it remains to show that such a policy satisfies the government budget given the same expenditure $G$, the same all time transfer $\chi$ and the same proportional taxes $\tau_a$ and $\tau_l$. In period 0:

$$\begin{aligned} G + r_0B_0 + TR_{0,new} &= B_{1,new} - B_0 + \tau_l w_0 L_0 + \bar{\tau}_a r_0 \widehat{A}_0 \\ G + r_0B_0 + TR + \Delta b &= B_1 + \Delta b - B_0 + \tau_l w_0 L_0 + \bar{\tau}_a r_0 \widehat{A}_0 \end{aligned}$$

In period $t > 0$:

$$\begin{aligned} G + r_t B_{t,new} + TR &= B_{t+1,new} - B_{t,new} + \tau_l w_t L_t + \bar{\tau}_a r_t \widehat{A}_{t,new} + T_{t,new} \\ G + r_t(B_t + \Delta b) + TR &= B_{t+1} + \Delta b - B_t - \Delta b + \tau_l w_t L_t + \bar{\tau}_a r_t(\widehat{A}_t + \Delta b) \\ &\quad +(1 - \bar{\tau}_a) r_t \Delta b \end{aligned}$$

Thus we have shown that in the model with natural borrowing limits there is Ricardian equivalence.

Suppose however we have instead an ad hoc borrowing limit which is binding for some households. In this case everything is the same except that for some agents savings are determined by $a'^* = \underline{a}$ and their Euler equation is not satisfied:

$$\frac{\beta \sum_{\epsilon_{t+1}} \pi(\epsilon_{t+1}|\epsilon_t) u_c(c^*_{t+1}, l^*_{t+1})}{u_c(c^*_t, l^*_t)} < \frac{1}{1 + \bar{r}_{t+1}} \text{ for agents with } a'^* = \underline{a} \tag{A.8}$$

Now we can show by contradiction that Ricardian equivalence does not hold. Suppose Ricardian equivalence would hold, then the borrowing constrained households would have to save $a'^*_{new} = \underline{a} + \Delta b$ and choose the same path for consumption and leisure. But if they choose the same path for consumption and leisure their Euler equation is still not satisfied, as shown by equation (A.8) above. But if the Euler equation doesn't hold for agents with $a'^*_{new} = \underline{a} + \Delta b$ it means they would like to go more into debt and they can do so as $a'^*_{new} = \underline{a} + \Delta b > \underline{a}$. Consequently they will not save $a'^*_{new} = \underline{a} + \Delta b$. There is a contradiction. We have thus shown that Ricardian equivalence cannot hold in this case.

### A.8.3 Definition of the Consumption Equivalent Welfare Change

The consumption equivalent welfare change for the average household is defined as the percentage change in consumption that the household must incur in the old situation in order to be indifferent between staying in the old stationary equilibrium and jumping to the new stationary equilibrium. Let the old stationary equilibrium be denoted by the subscript 0 and be characterized by a (detrended) debt level $\tilde{b}_0 = \frac{B_0}{Y_0}$ and a resulting density $\theta_0$. In our computations this point of comparison will always be the benchmark equilibrium with $\tilde{b}_0 = \frac{2}{3}$. Let the new stationary equilibrium be denoted by the subscript 1 and characterized by the debt level $\tilde{b}_1 \neq \tilde{b}_0$ and a resulting density $\theta_1 \neq \theta_0$. The consumption equivalent welfare change for a household with assets $\tilde{a}$ and income state $\epsilon$, $x_{0\to 1}$ is the percentage change in consumption in situation 0 that makes the household indifferent between staying in 0 and going to 1:

$$
\begin{aligned}
&\int W(\tilde{a},\epsilon;\theta_0,x_{0\to 1})d\theta_0(\tilde{a},\epsilon) = \int W(\tilde{a},\epsilon;\theta_1)d\theta_1(\tilde{a},\epsilon)\\
&\int E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{\left((c(\tilde{a},\epsilon;\theta_0)\cdot(1+x_{0\to 1}))^{\eta}\, l(\tilde{a}_t,\epsilon_t;\theta_0)^{1-\eta}\right)^{1-\mu}}{1-\mu} d\theta_0(\tilde{a},\epsilon) =\\
&\int E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{\left(c(\tilde{a},\epsilon;\theta_1)^{\eta} l(\tilde{a},\epsilon;\theta_1)^{1-\eta}\right)^{1-\mu}}{1-\mu} d\theta_1(\tilde{a},\epsilon)
\end{aligned}
$$

Solving this equation for $x_{0\to 1}$ we obtain:

$$
\begin{aligned}
x_{0\to 1} &= \left( \frac{\int E_{t=0} \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{\left(c(\tilde{a},\epsilon;\theta_1)^{\eta} l(\tilde{a},\epsilon;\theta_1)^{1-\eta}\right)^{1-\mu}}{1-\mu} d\theta_1(\tilde{a},\epsilon)}{\int E_{t=0} \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{\left(c(\tilde{a},\epsilon;\theta_0)^{\eta} l(\tilde{a}_t,\epsilon_t;\theta_0)^{1-\eta}\right)^{1-\mu}}{1-\mu} d\theta_0(\tilde{a},\epsilon)} \right)^{\frac{1}{\eta(1-\mu)}} - 1\\
&= \left( \frac{\int W(\tilde{a},\epsilon;\theta_1)d\theta_1(\tilde{a},\epsilon)}{\int W(\tilde{a},\epsilon;\theta_0)d\theta_0(\tilde{a},\epsilon)} \right)^{\frac{1}{\eta(1-\mu)}} - 1
\end{aligned}
$$

Logically, if $x_{0\to 1}$ is positive (negative) the average household would absent the compensation (not) prefer to be in the new situation. Thus a positive (negative) consumption equivalent welfare change means that the new debt level is (not) preferred in terms of welfare of the average household.

### A.8.4 Numerical Algorithm

The general algorithm to find the recursive general equilibrium is similar to Ábrahám and Cárceles-Poveda (2010). Given a level of government debt, $b$, a tax rate for labor income, $\tau_l$, a tax rate for capital income, $\tau_a$ and a guess for the borrowing limit, $\underline{a}$, we search for the policy functions for consumption, leisure and next periods assets (savings), $c(.)$, $l(.)$ and $a^{'}(.)$ by using a policy function iteration method. More precisely, we use an endogenous grid point method, commonly used in the literature, which we extend to include endogenous labor. Then we compute the implied wealth density function using interpolation methods. Given the wealth densities we obtain a new equilibrium aggregate capital stock which implies a certain equilibrium interest rate and wage rate, as well as a new tax rate. Note that we either change the capital income tax or the labor income tax depending on the experiment. (Denote the respective tax rate simply as $\tau$ in what follows). Then we find the final equilibrium in two steps first iterating simultaneously over the aggregate capital stock and the tax rate we want to change and then in a further outer loop iterating over the borrowing limit (if endogenous). More precisely we take out the following steps to find the equilibrium given a certain level of government debt:

**Loop to find borrowing limit** Given a certain level of government debt, $b$, guess an initial vector for the borrowing limit $\underline{a}^0$. Denote iterations in this loop by $p$.

**Loop to find interest rate and tax rate** Denote iterations in this loop by $q$.

**Step 1** Given a borrowing limit guess initial values for the tax rate, the aggregate labor supply and the aggregate capital stock: $[\tau^0, L^0, K^0]$. Those values for the aggregate capital stock and labor supply imply a certain interest rate $r^0$ and a certain wage rate $w^0$.

**Step 2** Guess initial values for the policy functions $c^0(.)$, $l^0(.)$,$a'^0(.)$. Now comes an inner loop to find the new policy functions. Denote iterations in this loop by $n$.

**Step 3** For each iteration $n$ given $[w^{q-1}, r^{q-1}, \tau^{q-1}, \underline{a}^{p-1}]$ use the guesses, $[c^{n-1}(.), l^{n-1}(.), a'^{n-1}(.)]$ to compute new policy functions that satisfy the first order conditions of the households, $[c^n(.), l^n(.), a'^n(.)]$.

**Step 4** Find the associated density function over assets and income states.

**Step 5** Given the density, compute the new aggregate capital stock $K^q$ and the new aggregate labor supply $L^q$, which imply a new wage rate $w^q$, a new interest rate $r^q$ and a new tax rate $\tau^q$. Then repeat Step 2-5 until we find the equilibrium interest rate, wage rate and tax rate given the guess for the borrowing limit.

**Step 6** Use the policy functions at the final equilibrium to compute the value of autarky $V^p$ and the value of being in the financial market arrangement $W^p$, given the previous value for the borrowing limit $\underline{a}^{p-1}$.

**Step 7** Compute the new borrowing limit, $\underline{a}^p$, by tightening them if $V^p > W^p$ and loosening them if $W^p > V^p$. Then repeat Step 1-7 until the equilibrium borrowing limit is found.

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# Appendix B

# Homeownership, Mobility and Local Income Redistribution


## Paper Summary

Regions are characterized by different homeownership rates. Homeowners and renters differ in their mobility costs, renters having lower mobility costs. This paper analyses how the presence of those different types of households affects income sorting and tax differences between local jurisdictions. To this aim we analyze a model of local income redistribution with mobile (renters) and immobile (homeowners) households. Linear income taxes finance a lump sum transfer. Policies are determined endogenously through voting. In such a framework if there are no or only few homeowners no income sorting equilibrium exists. Above a certain threshold for the homeownership rate we find an inverted U-shaped relationship between tax differences and homeownership rates, tax differences between jurisdictions being highest for intermediate homeownership rates.

## B.1 Introduction

In democratic countries citizens themselves exert an important influence on local fiscal policy through the elections of political representatives. If those countries are federalistically organized a central aspect is the fiscal autonomy of local jurisdictions. Clearly, in this case, the mobility behaviour of citizens has important effects on local fiscal policies. Income *sorting* or *stratification* patterns where households locate into different jurisdictions according to their income class are often observed empirically in democratic federalistic countries (Pommerehne et al., 1996; Feld and Kirchgässner, 2001; Bakija and Slemrod, 2004; Schmidheiny, 2006a). Rich households typically live in jurisdictions with low tax rates and high house prices whereas poor households live in areas with high taxes and low house prices. Thus it is important to understand the factors determining income sorting, because it has an influence on the daily lives of citizens in their respective jurisdictions, for example, on housing rents, taxation and redistributionary policies at the local level.

The literature on income sorting analyzes how different factors determine interjurisdictional income sorting equilibria. For example, Hansen and Kessler (2001) focus on the influence of jurisdiction sizes and Kessler and Lülfesmann (2005) and Schmidheiny (2006b) on the influence of heterogeneity in preferences for public goods and/or housing. Another potentially important factor are differences in mobility costs arising, for example, from the homeownership status of a household. The contribution of this paper is to investigate theoretically the influence of this factor on local income sorting equilibria.

We begin by presenting some empirical facts about income sorting and homeownership to motivate our analysis. Empirical studies show that homeowners are less mobile than renters (Barceló, 2006; Helderman et al., 2004; Munch et al., 2006; Ioannides and Kan, 1996). Analyzing tax differences between communities in Swiss Cantons, we find evidence for a non-linear relationship between those tax differences and the homeownership rate. In our view, this could be due to differences in mobility costs of homeowners and renters. To illustrate the influence of differences in mobility costs on local income sorting equilibria we then analyze a stylized set-up where we differentiate between two types of households: *Homeowners* with infinite mobility costs and *renters* with zero mobility costs. Consequently homeowners will not move at all whereas renters are perfectly mobile. We focus on a purely redistributive model where a distortive tax on income finances a lump sum transfer (Goodspeed, 1986, 1989; Hansen and Kessler, 2001). Hence, higher taxation means more redistribution but is costly because of distortions. For redistribution to be an interesting political outcome households have to differ in terms of income. Thus, some households are net contributors whereas others are net-beneficiaries. To be able to obtain analytical results we focus the analysis on the case of two jurisdictions and assume income to be distributed lognormally among households irrespective of their homeownership status. Furthermore to concentrate on the effect of mobility costs we abstract from different jurisdictional sizes and assume that jurisdictions are of equal size.

Households take decisions in two stages: First a household decides in which jurisdiction it wants to live considering expectations about the amount of redistribution that will be implemented in the respective jurisdictions. Second, each household has the possibility to elect a local political candidate with a certain electoral program of redistributive taxation through majority voting. The median voter theorem implies that the policy that will be implemented is the policy preferred by the median voter. When households differ with respect to income it clearly depends on the intrajurisdictional income distribution and resulting identity of the median voter how much redistribution will result in equilibrium. When the median voter is relatively poor (rich), redistribution will be high (low). Clearly, if part of the population is not mobile, this changes the amount of equilibrium redistribution relative to the case where everyone is mobile, as immobile households change the voting outcome by their presence.

We find the following results: First, if there are no or only few homeowners no income sorting equilibrium exists. The reason is that the income distribution is typically skewed to the right, meaning that there are much fewer very rich than very poor persons. If everyone was mobile and we had income sorting, the richest half of the population would live in one jurisdiction (referred to as *rich* jurisdiction in the following) and the poorest half in the other (referred to as *poor* jurisdiction). The median voter in the rich jurisdiction would however be poor, relative to the average person in that community, compared to the median voter in the poor jurisdiction. More precisely, the mean-to-median ratio (to which we refer to in the following by the term *m-to-m inequality*) in the rich jurisdiction would be higher than in the poor jurisdiction and thus taxes would be higher in the rich jurisdiction. Therefore the rich and also the poor would not like to live in such a segregated way in the first place, so that no income sorting equilibrium exists for completely mobile households and equally sized jurisdictions.[1] However, we show in this paper that, if there is a certain number of immobile homeowners in the population, an income sorting equilibrium can exist even with equally sized jurisdictions. Intuitively, the presence of immobile homeowners has a exarcerbating effect on m-to-m inequality in the poor jurisdiction. For this reason, tax rates are relatively higher in the poor jurisdiction with more immobile homeowners. With enough immobile homeowners the tax rate in the poor jurisdiction will even overtake the one in the rich jurisdiction. We thus find a threshold homeownership rate (or number of immobile households in the population) from which on an income sorting equilibrium exists.

Our second result is that from this threshold onwards there is an inverted

[1] This result was already emphasized by Hansen and Kessler (2001). They show that the presence of differently sized jurisdictions is necessary to induce income sorting. The presence of a very small jurisdiction to accomodate the richest households is crucial. They characterize the threshold value for the smallest jurisdiction such that an income sorting equilibrium exists. Extending their analysis we show that even with equally sized jurisdictions income sorting can arise due to the presence of immobile households (homeowners).

U-shaped relationship between homeownership rates and tax rates. For a range of moderate homeownership rates the difference between tax rates increases with the homeownership rate. Intuitively, as long as m-to-m inequality between homeowners is lower than m-to-m inequality between the richest renters, a higher presence of homeowners leads to lower m-to-m inequality (and thus higher taxes) in the rich jurisdiction relative to the poor jurisdiction. For high homeownership rates the difference between tax rates decreases with the homeownership rate. Intuitively, from some point onwards the tax rate in the poor and the rich jurisdictions will be more and more similar as the high number of homeowners with the same equilibrated income distribution makes the jurisdictions more and more similar in terms of m-to-m inequality. Finally, we find that an increase in overall inequality increases the range of homeownership rates where income sorting arises (or equivalently decreases the threshold homeownership rate).

Our paper aims at contributing to the literature on multijurisdictional models of local income redistribution with endogenous policy determination by emphasizing the impact of household heterogeneity in terms of mobility behaviour on the income sorting equilibrium. We argue that homeowners are less mobile than renters and focus on including those two groups. Epple and Romer (1991) also differentiate between renters and homeowners. They propose a similar redistribution model but where redistribution is financed via property taxes. The difference between renters and homeowners in their model is that tax capitalization in house prices only affects homeowners and not renters. In contrast, we focus on differences in mobility behaviour of those two groups. Hansen and Kessler (2001) analyze a similar model to explain the existence and size of tax havens. They point out to the fact that tax havens empirically are rather small countries. They show that a jurisdiction of a very small size is needed which can accomodate the most affluent households for an income sorting equilibrium to exist. In contrast, we abstract from size differences and focus on the impact of mobility differences. In reality, it is probably the interaction of both those factors (and possibly other factors as well) which determine the existence of an income sorting equilibrium and the exact location of each income group. Other factors determining income sorting are analyzed, for example, by Kessler and Lülfesmann (2005) (heterogeneity of preferences for public goods) or Schmidheiny (2006b) (heterogeneity of preferences for public goods and housing).

In addition, our paper can also contribute to the discussion if income redistribution is feasible at all on a local level in a federalistic country where citizens are mobile (Wildasin, 1991; Brueckner, 2000; Lee, 1998, 2007). Perfect income sorting puts important constraints on local income redistribution because in this case income can only be redistributed within income groups. However perfect income sorting is rarely observed empirically. Similar to Schmidheiny (2006b) although for different reasons our model implies imperfect income sorting. In this case, more redistribution is feasible.

The remainder of this paper is structured as follows: Section 2 displays empirical evidence that motivates our study. In Section 3 we discuss the related literature. In Section 4 we present our theoretical model and in Section 5 we discuss the equilibrium results. Finally, Section 6 offers some concluding remarks.

## B.2 Income Sorting and Homeownership: Some Empirical Facts

In this paper, we emphasize that homeowners are less mobile than renters. Empirical evidence indeed shows that this is the case. For example, Barceló (2006) finds that residential mobility is lower among homeowners than among renters in Europe from 1994 to 1998, Helderman et al. (2004) find the same for the Netherlands over the period 1981 to 1998, Munch et al. (2006) for Danemark over the period 1997 to 2000 and Ioannides and Kan

(1996) mention that this holds also for the US in 1987. There can be different reasons for this: (1) Homeowners face higher transaction costs than renters when moving. For example, transaction costs for selling and buying houses are higher than for changing renting contracts. (2) Houses often pass from parents to children in the form of bequests. (3) There are changes in the life-cycle component to individual mobility that go in the direction of slowing down changes in homeownership. According to a survey of many developed economies by Scanlon and Whitehead (2004), if younger households enter homeownership they do it at a later stage in life than previously. This is in line with the results of Mulder and Wagner (2001) showing that British and US families are likely to postpone family formation depending on the availability of housing. (4) Synergy effects when owning and investing in a house at the same time could be responsible for lower mobility of homeowners. For example, Brueckner and Joo (1991) explain that homeowners have a longer time horizon because they often consider their houses as investment assets as well as living facilities. If a house was bought as a long term investment to save for retirement and the owner wanted to profit from synergy effects of owning the house (for example, because an owner takes better care of the house than a renter would do), this would supposedly reduce the mobility of the homeowner until he reaches the retirement age.

Another important assumption of our model is that the overall homeownership rate in a region with multiple jurisdictions is taken to be exogenously given, i.e. independent of income sorting and resulting tax or rent differences in that region. Empirically, there is no clear consensus what determines the homeownership rate in a region. Religious affiliations seem to play a role, as emphasized, for example, by Keister (2003) or tax exemptions for homeownership on the national level. If homeownership rates on the regional level were caused by such factors, this would still be consistent with our approach as long as those factors are not in turn strongly interacting with income sorting or local income taxation.

Furthermore overall homeownership rates seem to change only slowly over time. A selection of a number of countries from Atterhög (2005) reveals overall stable homeownership patterns (see Figure B.1). Therefore we take the view that a tendency to homeownership is related to historical or cultural factors and thus exogenous to the income sorting equilibrium.

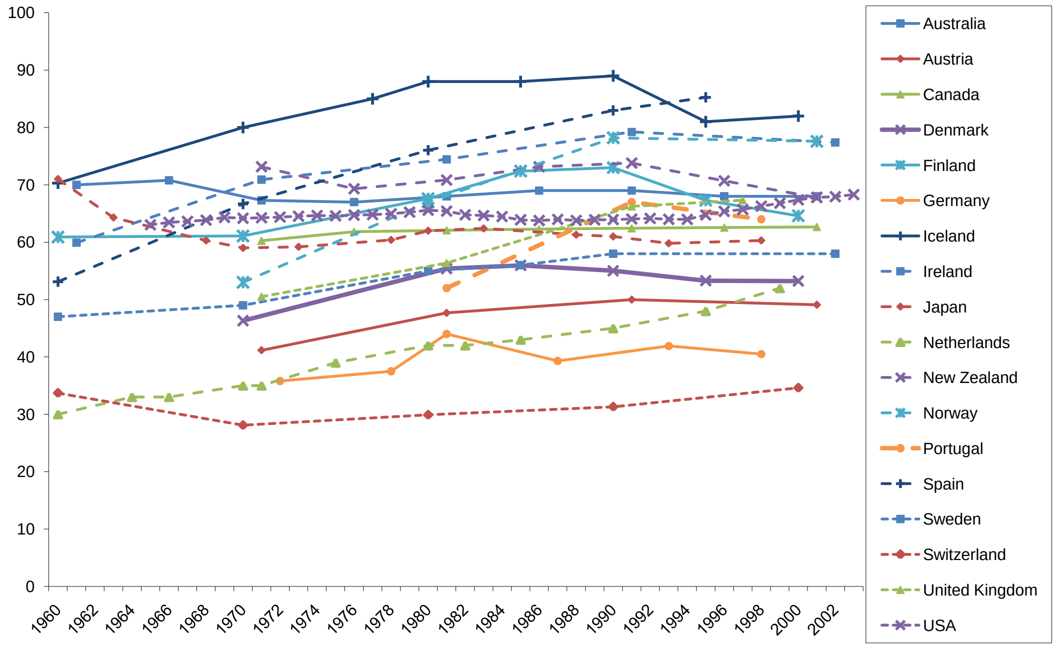


**Figure B.1:** Homeownership Rates Over Time in OECD Countries

Our model aims at explaining tax-induced income sorting across jurisdictions. Empirically, tax migration and sorting by income groups and effects on redistribution are frequently observed. Schmidheiny (2006a) shows that in Switzerland high income households are more likely to choose low tax jurisdictions. Using data from US federal states, Bakija and Slemrod (2004) find that wealthy retirees try to avoid high state taxes by changing their state of residence. Feld and Kirchgässner (2001) find a negative relationship between tax rates and the share of rich households. Using data from Swiss cantons and main cities, they regress the share of various income classes on income tax rates. Pommerehne et al. (1996) find some evidence for tax haven Cantons and tax-induced migration in Switzerland, but they conclude that a "race to the bottom" is not a concern.

Looking at Switzerland for empirical evidence on tax-induced income sorting may be case point. Real-world conditions in Switzerland are probably closest to most theoretical models, including ours. Many jurisdictions (the Cantons or the communes) decide independently and democratically on taxation and on the provision of public services. It is therefore an ideal laboratory for an empirical test of diverse theoretical predictions. When taking the coefficient of variance in communal tax rates as a measure for tax differences between communities within a canton, we find a positive relationship between income inequality (measured by the Gini coefficient) and tax differences as shown in Figure B.2. A higher overall income inequality seems to be linked to higher differences in communal tax rates within a region.

Taking the overall homeownership rate of a canton as a measure of mobility, we find a non-linear inverted U-shaped relationship between mobility and the coefficient of variance in communal tax rates as shown in Figure B.3. Tax differences within a region (canton) are less pronounced for high mobility (low homeownership rates) up to a certain threshold and falls again for low mobility (high homeownership rates).

## B.3 Related Literature

In his seminal paper Tiebout (1956) argues that a decentralized system of taxation and public goods provision allows citizens to "shop among jurisdictions" according to their preference for public goods. This idea is known as "Tiebout sorting". Empirically, Oates (1969) finds evidence for Tiebout

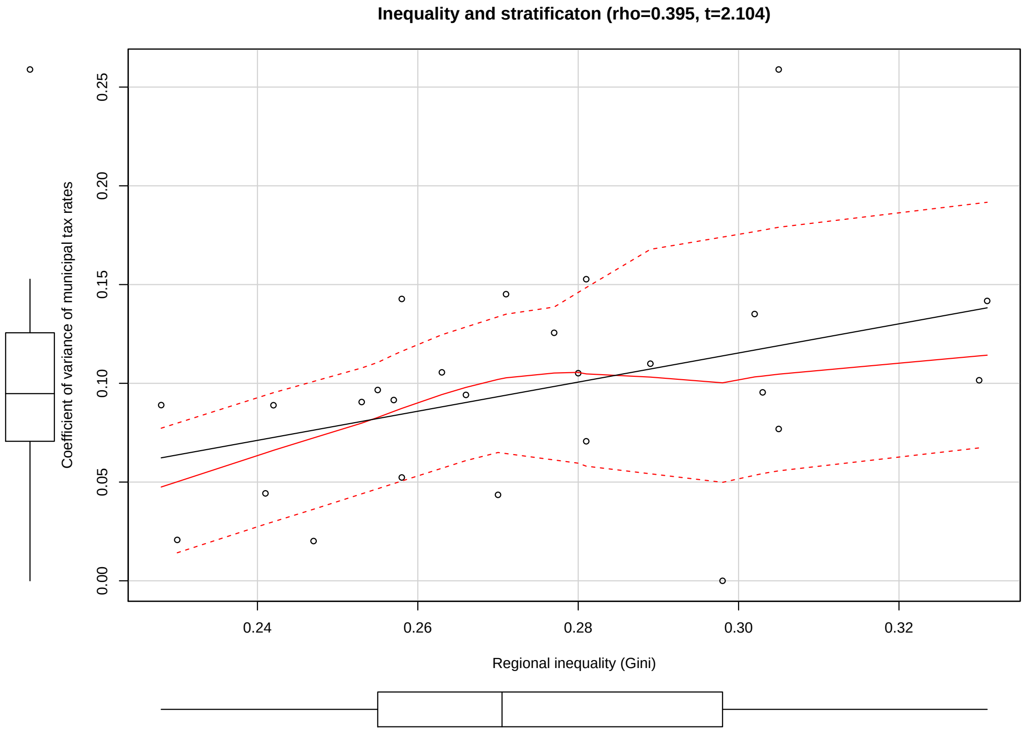


**Figure B.2:** Variance in Intra-Regional Tax Rates and Income Inequality

sorting, whereas Pollakowski (1973) shows that his initial results are not very robust. More recently Rhode and Strumpf (2003) find that heterogeneity of public goods across communities decreases over time which goes against the predictions of an extended Tiebout model. The later theoretical literature therefore deviates explicitely from the assumptions of the Tiebout model to test its robustness (Westhoff, 1977, 1979; Rose-Ackerman, 1979; Bucovetsky, 1982).

Subsequent theoretical work combines an explicit voting process with a migrational decision à la Tiebout to obtain a general equilibrium approach (Epple et al., 1984, 1993; Nechyba, 1997; Epple and Platt, 1998). In this approach, individuals typically move to a preferred jurisdiction in a first stage and there is majority voting over fiscal policy and taxes in a second stage. When moving individuals foresee the outcome of majority voting in the sec-

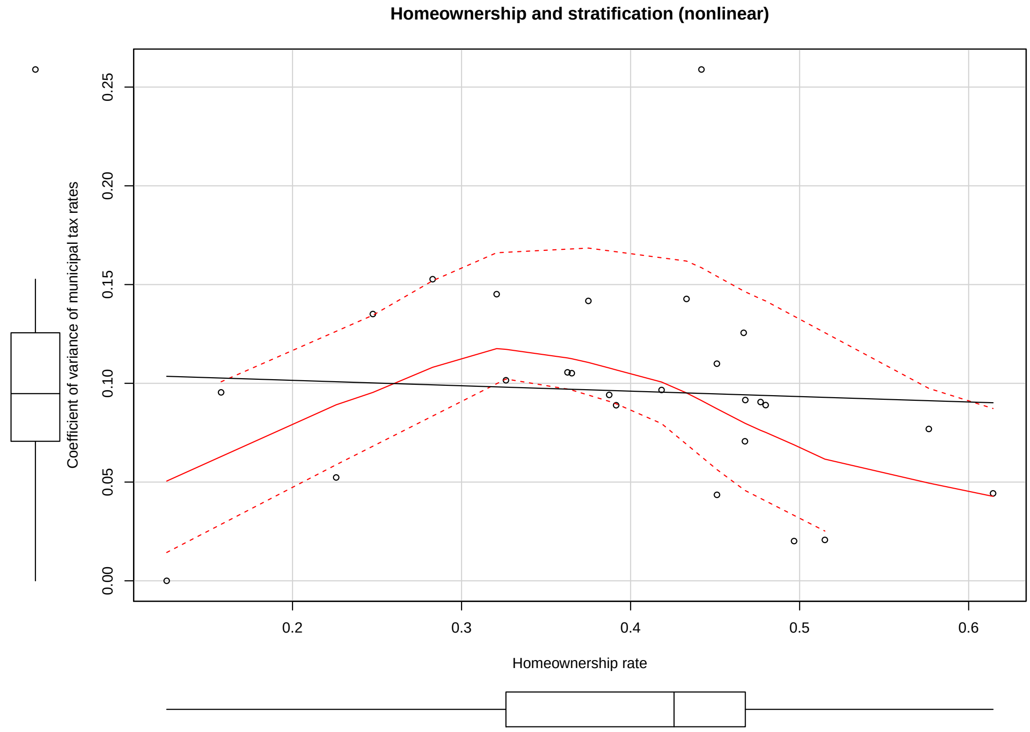


**Figure B.3:** Variance in Intra-Regional Tax Rates and Homeownership

ond stage, such that once an equilibrium is reached, they have no further incentive to move. The present paper follows such an approach. However, different than most of the previous authors who analyze models of property taxation which is the dominant form of local public finance in the US, we consider a model of income taxation.

Despite the frequent focus on property taxation, there are some notable exceptions that analyze general equilibrium models of local public good provision with income taxation (Goodspeed, 1986, 1989; Hansen and Kessler, 2001; Kessler and Lülfesmann, 2005; Schmidheiny, 2006b). Those contributions all have a slightly different focus to ours in the sense that they do not analyze the influence of different degrees of mobility on income sorting. Goodspeed (1986) derives the conditions for existence of an equilibrium in a metropolitan model with income taxation, congestible public goods and a housing market. He also estimates the welfare loss from changing from a head tax to a proportional income tax. However, he does not consider the influence of different mobility patterns.

Epple and Romer (1991) analyze the influence of a special aspect of mobility in a setting with property taxation. More specifically they differentiate between two main cases: (1) a model where there are only renters who rent houses from absentee landlords and (2) a model with homeowners who consider also the capital gains or losses they will incur as a result of a change in the net-of-tax price of housing induced by a change in the level of redistributive taxation. The conditions for the existence of an equilibrium of the model are the same in both cases, but an owner with a given endowed income will prefer a lower level of redistributive taxation than a renter with the same income. Hansen and Kessler (2001) focus on explaining the existence of tax havens depending on the size of a country (or jurisdiction). They show that if there is a jurisdiction of a very small size which can accomodate the most affluent individuals, an income sorting equilibrium can arise. In Hansen and Kessler (2001) the decisive characteristic of a jurisdiction which leads to income sortingof individuals with respect to income is the difference in country (or jurisdictional) size, whereas in our setting the fraction of the non-mobile population plays a similar crucial role. Alternatively, Kessler and Lülfesmann (2005) show that income sorting equilibria can be due to different preferences for public goods. Finally, analyzing a richer model where individuals differ in both income and preferences for housing, Schmidheiny (2006b) derives imperfect income segregation. All these contributions, like ours, show the possible existence of income sorting equilibria. However the approach presented in this paper is different as it is the fraction of immobile individuals which matters for income sorting to occur. This paper consequently extends existing work on income sorting equilibria in models with income taxation.

Similarly to Epple and Romer (1991) and Hansen and Kessler (2001) we analyze a purely redistributive setting without considering a public good which could enter the utility function explicitely. Grossmann (2002) argues that the distinction between a purely redistributive setting versus a public goods setting matters in models of majority voting because the consequences for the nature of the link between inequality and redistribution (or the size of government) differ between those settings. In a redistributive setting higher income inequality leads to more redistribution in equilibrium, whereas in a public goods setting a higher income inequality may lead to less redistribution. Empirically, the relationship between income inequality and redistribution has been investigated by many authors with rather inconclusive results (Perotti, 1994; Persson and Tabellini, 1994). Bénabou (2002) presents a review of the results of this strand of literature and finds that "the results are rather disappointing: the effect of income distribution on transfers and taxes is rarely significant, and its sign varies from one study or even one specification to another."[2] However, some studies find evidence in favor of the redistributive setting (Milanovic, 2000; Perotti, 1996). Being aware of the criticism we still opt for a redistributive setting, as our main interest is to assess the implications of mobility on taxation and on opportunities for income redistribution via grants.

We look at jurisdictions in a metropolitan area centered around a big city, where all individuals go to work in the same place but live in different jurisdictions. Therefore, we neglect possible effects coming from local productivity differences which would be important when looking at jurisdictions on a higher level such as countries. (Productivity differences are taken into account in models of spatial asset pricing, see e.g. Ortalo-Magné and Prat, 2008). Furthermore, we look at a static model which does not allow for growth and human capital accumulation (effects which are analyzed in recent contributions such as Bénabou, 1996a,b,c; Glomm and Lagunoff, 1999;

[2]See Bénabou (2002), p. 24.

Fernández and Rogerson, 1998 or Epple et al., 2009). Given our focus on redistributive government expenditures we leave such considerations out as they are especially interesting when analyzing schooling expenditures and public investments.

One main implication of our model is that in equilibrium individuals may be stratified with respect to income classes. As mentioned in the last section, some studies indeed provide empirical evidence for income sorting (see, for example, Bakija and Slemrod, 2004; Pommerehne et al., 1996; Feld and Kirchgässner, 2001; Liebig and Sousa-Poza, 2006 or Brülhart and Jametti, 2006). These studies deserve credit for showing that tax differences and income sorting occur, when jurisdictions enjoy fiscal autonomy and they show to what extent empirically individuals react to such tax differences by migration. However, they assume that tax rates are set exogenously by a local government (benevolent or not). It is not a priori clear how such results have to be interpreted given that in reality taxes are rather set endogenously, in most cases by a voting process where the residents have to decide for some political candidate or party who proposes a certain tax policy. Thus, more recently, a lot of authors attempted to structurally estimate or calibrate equilibrium models of local jurisdictions where voting is endogenous (see Epple and Sieg, 1999; Epple et al., 2001; Bayer et al., 2004; Bajari and Kahn, 2005; Sieg et al., 2002, 2004; Ferreyra, 2007; Ferreira, 2010 and for Switzerland Schmidheiny, 2006a). These contributions generally find strong evidence for income sorting between local jurisdictions as well.

The focus of our paper is on the influence of mobility patterns on taxation and redistribution. Therefore, we also contribute to the part of the literature which makes predictions about the influence of the degree of mobility on local taxation and the possibility of redistributive policies at the local level. In the "race-to-the-bottom" literature a higher mobility typically leads to lower taxes and less redistribution (see, for example, Brueckner, 2000). Empirical studies analyzing whether this breakdown of the welfare state really occurs yield different results. Brown and Oates (1987) and Feldstein and Wrobel (1998) find evidence for tax avoidance in the US similar to the one presented above and thus argue that redistribution should be taken out by the central government only. However, for Switzerland Feld (2000) finds that a considerable amount of redistribution takes place at the local level despite high mobility of tax payers and strong tax competition. Alternatively, Lee (2007) presents a political-support approach to redistribution in a federation, which leads to a different conclusion than the standard "race to the bottom" models. He finds that depending on the cost of housing (which is equivalent to mobility costs in his model), mobility may increase or decrease income redistribution. As will become clear from our analysis, our model in contrast predicts a non-linear relationship between redistribution and mobility, redistribution being highest for intermediate levels of mobility.

In the next section we will thus present a theoretical model where groups with different degrees of mobility have an influence on the income sorting equilibrium. The non-linear pattern between mobility and income sorting (redistribution) predicted by this model is consistent with the empirical facts about this relationship for Swiss Cantons presented in Section B.2. In addition the imperfect income sorting implied by the model is consistent with empirical findings about income sorting. The non-linear relationship between taxation and mobility could also explain the inconsistent empirical results presented in the last paragraph.

## B.4 The Model

We look at a system of integrated and independent jurisdictions denoted $j = 1, ..., J$ in an economy, all having the same size. The jurisdictions are politically independent in the sense that each jurisdiction determines fiscal policy in an autonomous way. We choose a setting similar to recent contributions on sorting such as Hansen and Kessler (2001) or Kessler and Lülfesmann (2005). But different from them, in our economy jurisdictions distinguish themselves only by the fraction of homeowners.

We develop a model of redistribution to analyze the interaction between homeownership, the economy's income inequality, and political decisions on redistributive taxes. The most important elements of the model are: (1) Individuals are heterogeneous with respect to income so that redistribution is an important political outcome. (2) A jurisdiction's budget must balance. (3) Migration is costless for the group of renters, i.e. in equilibrium renters must be unable to improve their positions by moving. (4) After settlement in a specific jurisdiction, political decisions are determined by majority voting.

There are two types of households in the economy: renters ($R$) and homeowners ($H$). To illustrate the mechanism we have in mind we assume those two types to differ with respect to mobility costs in a stark and stylized way: Renters face zero mobility costs so that they are completely mobile, whereas homeowners face infinite mobility costs so that they never move. We denote the number of renters in a jurisdiction $j$ by $\lambda_j^R$ and the number of homeowners by $\lambda_j^H$. All households, irrespective of their mobility cost type, earn an exogenous income $y^i$ which can be interpreted as location independent labor income.[3] Each household who does not own a house has to rent a unit of housing from competitive absentee landlords to gain the right to live and vote in a jurisdiction. Rents in jurisdiction $j$ are denoted as $r_j$. Housing demand is normalized to unity. The utility $V$ of an individual $i$ ($i \in [R, H]$) is assumed to be linear in consumption $c^i$ ($U^i = c^i$). All households in jurisdiction $j$ have to pay proportional income taxes $t_j$ on their income $y^i$ and

[3]We make this assumption, because we have in mind local jurisdictions, for example communities, in a metropolitan area, where there is a large number of commuters to a central city. Consequently, almost everyone in the community is working in the city and income is not affected by local conditions of the community.

receive a (basic income) grant $g_j$ from the local government.[4] The budget constraint of a renter $i$ in jurisdiction $j$ is thus

$$y^i + g_j = c^i + y^i t_j + r_j,$$

and indirect utility is equal to net income minus the rent

$$V^R(y^i, t_j, g_j, r_j) = (1 - t_j)y^i + g_j - r_j.$$

Homeowners, in contrast, do not need to pay rents as they live in their own house. The budget constraint of a homeowner $i$ in jurisdiction $j$ is thus given by

$$y^i + g_j = c^i + y^i t_j,$$

and indirect utility is consequently equal to

$$V^H(y^i, t_j, g_j) = (1 - t_j)y^i + g_j.$$

In the economy income is distributed across all individuals according to the distribution function $F(y)$ with density $f(y) > 0$ and support $[0, \max y]$. Households of each type $R$ and $H$ have the same ex-ante distribution of income. Denote the income distribution of households within a jurisdiction $j$ by $F_j(y)$. In every jurisdiction, the income distribution for homeowners is given by this ex-ante distribution since they face infinite mobility costs and do not move: $f_j^H(y) = f(y)$. The income distribution for renters for each jurisdiction $j$ instead emerges endogenously after migration and is denoted $f_j^R(y)$. We normalize each jurisdiction's size in the economy to unity. Let $\lambda_j = \frac{\lambda_j^H}{\lambda_j^H + \lambda_j^R}$ denote the percentage of homeowners in jurisdiction $j$ (called *homeownership rate* in the following) and thus the percentage of renters is simply given by $1 - \lambda_j$. In this paper, we assume $\lambda_j = \lambda$, i.e. equal homeownership rates in

[4]We might introduce local public good provision instead of pure redistribution. In this case the preference for public goods would play a role. Instead, we opt for a purely redistributive setting, because the focus of our paper is the influence of mobility and not the influence of different preferences for public goods.

each jurisdiction. While this may seem like a crude simplification, the general mechanism stays the same even if homeownership rates differ between jurisdictions except that there is an additional margin of variation.[5] From the viewpoint of renters, the homeownership rate $\lambda$ determines the size of a jurisdiction for them. Changes in average and median incomes depend on the part of the mobile (renters) as well as the immobile (homeowners) population. Therefore, in our setting, the homeownership rate has potentially different implications than a jurisdiction's size in the model of Hansen and Kessler (2001). To insure that rents in each jurisdiction of the economy are fully determined even when the population size in each jurisdiction is fixed, we suppose that competitive absentee landlords underbid each other in the jurisdiction with the lowest mean income. More specifically, in the jurisdiction with the lowest mean income rents must equal the break-even price $\underline{r}$ for absentee landlords.[6]

To focus on political motives for government and individual behavior, we abstract from allocative reasons for public spending and assume that jurisdictions raise income taxes for redistributive purposes only. The government has a redistribution policy in the sense that it confers the same basic income or grant $g_j$ to every individual in the jurisdiction. Proportional income taxes $t_j$ finance the grant $g_j$. This "ability to pay principle" is a common approximation of the progressive tax systems in use. A jurisdiction's budget must balance and thus the budget constraint is

$$(t_j - \frac{1}{2}t_j^2) \int_0^{\max y} y dF_j = g_j.$$

where $F_j$ denotes the endogenous measure of individuals with income $y$ living in jurisdiction $j$ such that average income in jurisdiction $j$ can be expressed as $\bar{y}_j = \int_0^{\max y} y dF_j$. The budget constraint equates local income tax revenues

[5]Results for the equilibrium analysis with different homeownership rates can be obtained from the authors upon request.

[6]Technically we could also assume that there is an oversupply of houses.

minus costs of raising public funds. The term $(t_j - \frac{1}{2}t_j^2)$ outlines a concave per capita Laffer-curve. Taxation is costly with costs taking the form $t_j^2/2$ for simplicity. We assume that the jurisdictions all have the same efficiency in redistribution, i.e. the same costs of taxation.[7] Because of the binding budget constraint we can solve for the grant $g_j$, which is then determined by the chosen tax rate

$$g_j = \left(t_j - \frac{1}{2}t_j^2\right)\bar{y}_j. \qquad \text{(B.1)}$$

## B.5 Equilibrium Analysis

The equilibrium of the model is determined in two steps:

1. Renters choose the jurisdiction to live in which results in the locational equilibrium.

2. All households in a jurisdiction cast their votes which results in the voting equilibrium.

When backward solving the model we look at the voting equilibrium first. Given the conditions resulting from the voting equilibrium we analyze the locational equilibrium.

### B.5.1 Definition of the Equilibrium

Every voter $i$ maximizes his indirect utility with respect to the tax rate and the basic income grant given the budget constraint. Rents are not directly relevant to the voting decision and can thus be left out at this step, i.e.

[7] As incomes are exogenous and pure redistribution is analyzed, the absence of costs of taxation would allow for arbitrarily high tax rates. Deadweight losses of distortive taxation could also be introduced more generally by endogenous labor supply. This would mainly complicate the analysis without changing the qualitative results and providing additional insights.

homeowners and renters will not differ in their voting decision given their income:

$$\max_{t_j}(1 - t_j)y^i + (t_j - \frac{1}{2}t_j^2)\bar{y}_j$$

The preferred tax rate of a household $i$ is thus given by: ${t_j^i}^* = 1 - \frac{y^i}{\bar{y}_j}$. In the unique majority voting equilibrium the tax choice of the median voter is implemented, i.e.

$$t_j^* = 1 - \frac{y_j^m}{\bar{y}_j}, \tag{B.2}$$

where $y_j^m$ is the income of the median voter in jurisdiction $j$. The level of redistribution in equilibrium is thus given by $g_j^* = \frac{1}{2}\frac{\bar{y}_j^2 - \left(y_j^m\right)^2}{\bar{y}_j}$. Redistribution will be higher in jurisdictions with high mean income and lower in jurisdictions with a high median income. The political choice by the median voter determines the fiscal package of taxes and grants $(t_j^*, g_j^*)$. To facilitate the explanations in the rest of the paper let's define the specific concept of inequality encountered in this voting model (mean-to-median inequality):

**DEFINITION B.3.** *Define* m-to-m inequality *as the ratio of the mean to the median in a jurisdiction:*

$$\textit{m-to-m inequality}_j = \frac{\bar{y}_j}{y_j^m}$$

Note that the higher the m-to-m inequality in a jurisdiction, the more redistribution there is and the higher taxes are in equilibrium (see equation (B.2)).[8]

Given the equilibrium tax rate, we can determine the locational equilibrium, i.e. how the different types of households locate over jurisdictions. In a locational equilibrium each type of renter must be happy with his or her choice of jurisdiction and does not want to move given the choice of the other renters.

[8] Note also that this measure of inequality is positively correlated to other frequently used inequality measures, such as, for example, the Gini coefficient, under specific distributional assumption.

If household $i$ decides to settle in jurisdiction $j$ his or her utility must be greater or equal in jurisdiction $j$ than in any other jurisdiction denoted $-j$, i.e. $V(y^i, t_j, g_j, r_j) \geqslant V(y^i, t_{-j}, g_{-j}, r_{-j})$.

In summary, the equilibrium of the model is defined as follows:

**DEFINITION B.4.** *An equilibrium in the economy is defined as an income distribution of mobile individuals in each jurisdiction $f_j^{M^*}(y)$, a fiscal policy package $(t_j^*, g_j^*)$ and rental fees $r_j^*$ for all $j = 1, ..., J$, such that (1) no mobile individual with income $y^i$ living in $j$ has an incentive to move to another jurisdiction $-j$, that is, $V(y^i, t_j^*, g_j^*, r_j^*) \geqslant V(y^i, t_{-j}, g_{-j}, r_{-j})$ and (2) tax rates $t_j^* = 1 - \frac{y_j^m}{\overline{y}_j}$ reflect the median voter's choice in each jurisdiction.*

## B.5.2 A Symmetric Equilibrium

First, we show that independent of the homeownership rate $\lambda$, and the exact form of the income distribution, $F(y)$, there always exists a symmetric equilibrium in which all jurisdictions implement the same fiscal policy package, $(t_j, g_j)$, and rents, $r_j$, and average incomes, $\overline{y}_j$, are the same across all jurisdictions.

Suppose that the population is distributed symmetrically over all jurisdictions in the economy such that in each jurisdiction the local income distribution of mobile individuals is equal, i.e. $f_j^M(y) = f^M(y)$ for all $j = 1, ..., J$. This is equivalent to saying that median and average incomes are the same in all jurisdictions, independent of the homeownership rate $\lambda$. Therefore, tax rates and consequently also the level of redistribution will be the same in each jurisdiction. The clearing of renting markets implies that absentee landlords will set the same renting price denoted by $\underline{r}$. Therefore every mobile individual is indifferent between jurisdictions and thus the presumed distribution $f^M(y)$ is an equilibrium outcome. The following Proposition summarizes this finding:

**PROPOSITION B.1.** *Independent of the homeownership rate, $\lambda$, in the economy, a symmetric equilibrium with identical fiscal policies $(t_j, g_j) = (t^*, g^*)$, identical rents $r_j = \underline{r}$, and identical average incomes $\bar{y}_j = \bar{y}$ in all jurisdictions $j = 1, ..., J$ always exists.*

### B.5.3 An Asymmetric Equilibrium

Another possibility is that jurisdictions offer different tax-grant packages, i.e. $(t_j, g_j) \neq (t_h, g_h)$ for $j, h \in [1, ..., J]$ and $j \neq h$. If $(t_j, g_j) \neq (t_h, g_h)$ renters may sort themselves into different jurisdictions: a phenomenon referred to as *income sorting* in the literature. The existence of such an income sorting equilibrium depends on the homeownership rate, $\lambda$, as well as on the exact form of the ex-ante income distribution, $F(y)$.

To illustrate the possibility of income sorting, we focus for simplicity on the case of two jurisdictions with $(t_1, g_1) \neq (t_2, g_2)$. Suppose an income sorting equilibrium exists such that jurisdiction $j = 1$ is a low tax (wealthy) jurisdiction, whereas jurisdiction $j = 2$ is a high tax (poor) jurisdiction. Let us now characterize the resulting equilibrium. First, note that jurisdiction $j = 1$ must have a lower grant-rent differential than jurisdiction $j = 2$, i.e. $(t_1, g_1 - r_1) < (t_2, g_2 - r_2)$. This must be so, because otherwise all renters would like to live in the jurisdiction with low taxes and a high grant-rent differential.

Second, note that, as in equilibrium both jurisdictions must be populated and there is no extra-space, there must be a boundary renter with boundary income $\tilde{y}$ who is just indifferent between the two jurisdictions. If an income sorting equilibrium exists, all renters with income $y > \tilde{y}$ will live in the low tax (wealthy) jurisdiction $j = 1$ and all renters with $y \leq \tilde{y}$ will live in the high tax (poor) jurisdiction $j = 2$. In an income sorting equilibrium, the boundary renter with income $\tilde{y}$ is the renter with the lowest income in jurisdiction $j = 1$ whereas in jurisdiction $j = 2$ the renter with income $\tilde{y}$ has the highest income among renters.

Third, note that from (B.2) $\frac{y_1^m}{\bar{y}_1} > \frac{y_2^m}{\bar{y}_2}$ must hold to ensure $t_1 < t_2$. Or in other words, $\frac{y_1^m}{\bar{y}_1} > \frac{y_2^m}{\bar{y}_2}$ is a necessary condition for income sorting. To summarize this finding we formulate the following Lemma:

**LEMMA B.1.** *A necessary condition for an income sorting equilibrium is that m-to-m inequality is lower in the low tax jurisdiction than in the high tax jurisdiction.*

For the housing market to clear rents have to adjust such that exactly the fraction of wealthiest renters, $x^R = 1 - F(\tilde{y})$, wants to live in jurisdiction $j = 1$. Note that $x^R$ depends on the homeownership rate $\lambda$ in both jurisdictions as they determine how many people can live in each jurisdiction. In the case of two jurisdictions half of the renters are considered wealthy while the other half are considered poor. Consequenlty, the fraction of wealthiest renters equals $x^R = \frac{1}{2}$ and the boundary income equals the economy's median income, i.e. $\tilde{y} = F^{-1}(\frac{1}{2}) = y^m$.[9] Given the boundary income and a sorting equilibrium, the distribution of income, the mean and the median incomes in the two jurisdictions look as follows:

**LEMMA B.2.** *Let $\tilde{y}$ be the boundary income. In a sorting equilibrium the distributions of income in jurisdictions $j = 1$ and $j = 2$ are*

$$G_1(y) = \begin{cases} \lambda F(y) & y \leq \tilde{y} \\ (2-\lambda)F(y) - (1-\lambda) & y > \tilde{y} \end{cases}$$

$$G_2(y) = \begin{cases} (2-\lambda)F(y) & y \leq \tilde{y} \\ (1-\lambda) + \lambda F(y) & y > \tilde{y} \end{cases} .$$

[9]Expressed differently, the fraction of wealthiest renters is equal to the remaining space in the low tax jurisdiction divided by the number of renters from the economy's population, i.e. $x^R = \frac{1-\lambda}{2-\lambda-\lambda} = \frac{1}{2}$.

*The mean incomes in each jurisdiction are*

$$\bar{y}_1 = \lambda \int_0^{\max y} y f(y) dy + 2(1-\lambda) \int_{\tilde{y}}^{\max y} y f(y) dy, \tag{B.3}$$

$$\bar{y}_2 = \lambda \int_0^{\max y} y f(y) dy + 2(1-\lambda) \int_0^{\tilde{y}} y f(y) dy, \tag{B.4}$$

*and the median incomes in each jurisdiction are*

$$y_1^m = F^{-1}\left(\frac{1}{2}\frac{3-2\lambda}{2-\lambda}\right) \tag{B.5}$$

$$y_2^m = F^{-1}\left(\frac{1}{2}\frac{1}{2-\lambda}\right) \tag{B.6}$$

The proof of this Lemma is relegated to the Appendix.

As the mean and the median incomes in both jurisdiction depend on the homeownership rate in the economy, equilibrium tax rates also depend on $\lambda$, i.e. $t_1^*(\lambda) = 1 - \frac{y_1^m(\lambda)}{\bar{y}_1(\lambda)}$ and $t_2^*(\lambda) = 1 - \frac{y_2^m(\lambda)}{\bar{y}_2(\lambda)}$. By the government budget constraint the grant is determined as a function of $\lambda$, i.e. $g_j^*(\lambda) = \left(t_j^*(\lambda) - \frac{1}{2}t_j^{*2}(\lambda)\right)\bar{y}_j$. To see how rents are determined in this model consider the renter with boundary income $\tilde{y}$ who is indifferent beween jurisdiction $j = 1$ and $j = 2$, i.e.

$$\begin{aligned} V(t_1^*(\lambda), g_1^*(\lambda) - r_1, \tilde{y}) &= V(t_2^*(\lambda), g_2^*(\lambda) - r_2, \tilde{y}) \\ (1 - t_1^*(\lambda))\tilde{y} + g_1^*(\lambda) - r_1^* &= (1 - t_2^*(\lambda))\tilde{y} + g_2^*(\lambda) - r_2^*. \end{aligned}$$

Note that $r_2^*$ will be equal to the minimum rent $\underline{r}$ as absentee landlords only compete for renters in the high tax jurisdiction.[10] Thus, we have

$$r_1^* - \underline{r} = (t_2^*(\lambda) - t_1^*(\lambda))\tilde{y} + g_1^*(\lambda) - g_2^*(\lambda) > 0$$

[10]Note that technically there is an infinitesimally small oversupply of houses such that rents in jurisdiction 2 are determined, similar to the symmetric case. For notational simplicity we omit it from the formulas.

Different to the model of Hansen and Kessler (2001) where everyone is mobile and jurisdictions are of different sizes, for the model set-up here, there is no general result of existence of an income sorting equilibrium without specific assumptions on the ex-ante distribution of income. Furthermore, it is possible that tax rates become negative as the ex-post distribution of income is not unimodal anymore.[11] Due to this lack of general results, we will assume that ex-ante income is distributed lognormally in the next section and that parameters of the distribution are such that tax rates are non-negative. In this case, it is possible to show that if the homeownership rate, $\lambda$, is above a certain threshold, income sorting exists. The homeownership rate then plays a similar role as jurisdiction size in Hansen and Kessler (2001) determining the cases where income sorting exists.

Before we turn to the case of the lognormal distribution consider as an illustration the following simple example:

**EXAMPLE 1.** *Suppose $\lambda = 0$, meaning that everyone in the population is mobile and the distribution of income is uniform but different over three parts of the support, so that it is skewed to the right in a very stylized way:*

$$f(y) = \begin{cases} \frac{1}{2} & \textit{for } 0 \leq y \leq 1 \\ \frac{3}{8} & \textit{for } 1 < y \leq 2 \\ \frac{1}{8} & \textit{for } 2 < y \leq 3 \end{cases}$$

*There is some ex-ante m-to-m inequality in the economy as $\bar{y} = \frac{9}{8} > y^m = 1$. Suppose individuals were sorted by income classes, such that the richest half lives in jurisdiction 1 and the poorest half lives in jurisdiction 2. Figure B.4 Panel A illustrates a possible income sorting equilibrium in this case where*

[11] Decentralized political and locational choice may not produce a stable equilibrium solution. Sorting is not a general outcome of locational equilibrium models as contributions by Rose-Ackerman (1979), Epple and Platt (1998), Hansen and Kessler (2001) show. Westhoff (1977) constructs a model with a pure public good and shows that income sorting equilibria can exist. Nechyba (1997) states conditions for sorting equilibria to arise in models with property taxation.

*everyone is mobile. The first part of the Figure shows the densities of the population over income in the overall region which is at the same time the ex-ante density in each jurisdiction before migration. The boundary income is exactly* $\tilde{y} = y^m = 1$ *the dividing line between the two halves in the first subfigure. The two following subfigures show the densities that would result for the two communities in a possible income sorting equilibrium. In jurisdiction 1 the poor half of the population left and the rich half from the other community joined. The contrary holds for jurisdiction 2. This implies for median incomes, mean incomes and thus for the tax rates in both jurisdictions (for an illustration see the dashed lines in B.4 Panel A):*

$$\begin{aligned} y_1^m &= \frac{5}{3}, \bar{y}_1 = \frac{7}{4} \Rightarrow t_1^* = \frac{1}{21} \\ y_2^m &= \frac{1}{2}, \bar{y}_2 = \frac{1}{2} \Rightarrow t_2^* = 0 \end{aligned}$$

*Thus, the tax rate in jurisdiction* 2 *(poor jurisdiction) would be lower than in jurisdiction* 1 *(rich jurisdiction) if the population was sorted by income in this way . Intuitively the rich would like to go in jurisdiction* 2*, but if all the rich went in this jurisdiction the voting process would again yield a tax rate like in jurisdiction* 1 *now (jurisdiction are ex-ante identical!). The rich are unable to gather in one of the jurisdictions and agree about a low tax rate because the inequality between them is higher than the inequality between the poor. For* $\lambda = 0$ *an income sorting equilibrium cannot exist.*

*Now suppose that one third of the population is immobile* ($\lambda = \frac{1}{3}$)*. Figure B.4 Panel B illustrates this case. The first subfigure again shows the densities of households over income in the overall region which is at the same time the ex-ante density in each single jurisdiction before migration takes place. The only difference is that now some households in each income group are immobile (denoted by the red lines in the subfigures). Logically, as those households are immobile their densities do not change with migration (same pattern of red lines in each of the three subfigures). The boundary houshold is again*

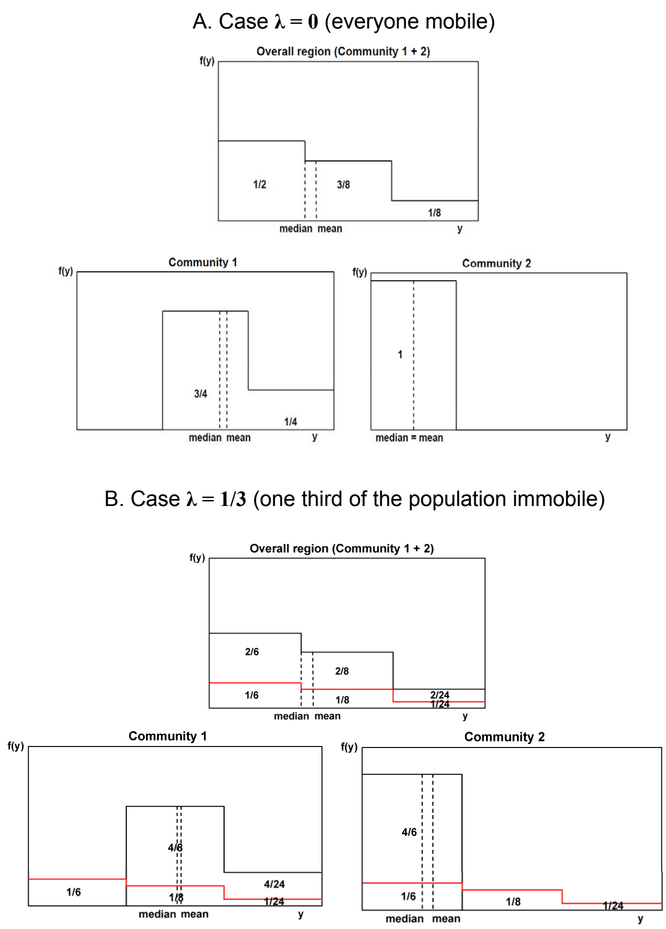


**Figure B.4:** Existence of Stratification Equilibria: A Simple Example

$\tilde{y} = y^m = 1$. *Now suppose again that the richest half moves to jurisdiction* 1 *whereas the poorest half moves to jurisdiction* 2 *with the only difference that now only the mobile population moves. After migration we would obtain for the medians and the means (for an illustration see the dashed lines in B.4 Panel B) :*

$$\begin{aligned} y_1^m &= \frac{23}{15}, \, \bar{y}_1 = \frac{37}{24} \Rightarrow t_1^* = \frac{1}{185} \\ y_2^m &= \frac{3}{5}, \, \bar{y}_2 = \frac{17}{24} \Rightarrow t_2^* = \frac{13}{85} \end{aligned}$$

*Tax rates in the rich jurisdiction are now lower than in the poor jurisdiction* *so that nobody wants to change his location ex-post. The resulting inequality after migration is lower in the rich jurisdiction and thus the tax rate will be lower there. Income sorting constitutes an equilibrium.*

This example illustrates the potential role that homeownership can play for the existence of income sorting equilibria. Hansen and Kessler (2001) noted that if jurisdictions do not have the same size and one jurisdiction is "small enough" then it is possible for the rich to gather there and agree on a tax rate that is lower than in the "big" jurisdiction, where m-to-m inequality will be higher. Here, we note that even if the jurisdicitons all have the same size, the fact that part of the population is not mobile can lead to income sorting.

Intuitively, if part of the population are homeowners, i.e. immobile by assumption (positive homeownership rate, $\lambda > 0$), this can increase m-to-m inequality in the jurisdiction where the poor decide to live relative to the case of no homeownership, as some rich homeowners will not move. For this reason, an income sorting equilibrium which implies a higher tax rate in the high tax relative to the low tax jurisdiction can arise with enough homeowners. However this mechanism depends on the exact assumption about the income distribution of households. The next section will thus discuss more in detail the existence of income sorting equilibria when income is distributed lognormally.

### B.5.4 Existence of an Income Sorting Equilibrium When Income is Distributed Lognormally

As already mentioned, it is not possible to show general results for existence without assuming a specific income distribution function. We propose to assume that income is distributed lognormally across households. This assumption implies that the distribution of income is bounded below and open above and that the biggest mass of households have relatively low incomes.[12] It turns out that the homeownership rate, $\lambda$, in the economy and the initial income inequality both have a distinguishible effect on the existence of a sorting equilibrium. Given a certain level of initial income inequality reflected by the parameter $\sigma$ for the lognormal distribution[13], a treshold $\tilde{\lambda}$ for homeownership rate is implicitely defined, such that from this threshold onwards an income sorting equilibrium exists. This is summarized by the following proposition:

**PROPOSITION B.2.** *Suppose income is lognormally distributed across individuals. For any given finite parameter $\sigma$, there is a unique threshold for the homeownership rate, $\tilde{\lambda}$, implicitly defined by*

$$
\begin{aligned}
t_2(\tilde{\lambda}) - t_1(\tilde{\lambda}) &= \frac{1}{\exp\left(\frac{\sigma^2}{2}\right)} \left( \frac{\exp\left(\sigma\sqrt{2}erf^{-1}\left(\frac{1-\tilde{\lambda}}{2-\tilde{\lambda}}\right)\right)}{\left[\tilde{\lambda} + 2(1-\tilde{\lambda})\Phi(\sigma)\right]} \right. \\
&\quad \left. - \frac{\exp\left(-\sigma\sqrt{2}erf^{-1}\left(\frac{1-\tilde{\lambda}}{2-\tilde{\lambda}}\right)\right)}{\left[\tilde{\lambda} + 2(1-\tilde{\lambda})\left(1-\Phi(\sigma)\right)\right]} \right) \qquad \text{(B.7)} \\
&= 0.
\end{aligned}
$$

*If the homeownership rate in the economy is lower than the threshold, i.e. $\lambda < \tilde{\lambda}$, there is no income sorting equilibrium. If the homeownership rate is higher than the threshold, i.e. $\lambda > \tilde{\lambda}$, an income sorting equilibrium exists, i.e. $t_2 - t_1 > 0$.*

(The proof is relegated to the Appendix). As an illustration, consider the case of $\sigma = 0.5$. Figure B.5, Panel A shows that, in this case, there is a threshold level for the homeownership rate $\tilde{\lambda} = 0.325$ from which on the

[12] As income distributions are usually skewed to the right the lognormal distribution has been found to approximate true income distributions quite closely and has been applied as a reasonably good characterization by other authors from the field (for example Epple and Romer, 1991 and Hansen and Kessler, 2001 among others).

[13] Different measures of inequality are increasing in $\sigma$ for the lognormal distribution, for example the variance of income, $Var(y) = e^{\mu+\sigma^2}(e^{\sigma^2} - 1)$ or the ratio of mean to median income, $\frac{\bar{y}}{y^m} = e^{\frac{\sigma^2}{2}}$.

difference between the tax rates is positive and thus a sorting equilibrium exists. At the left of this threshold a sorting equilibrium cannot exist regardless in which of the two jurisdictions the rich move as the jurisdiction in which the rich have moved will feature a higher m-to-m inequality and thus a higher tax rate.

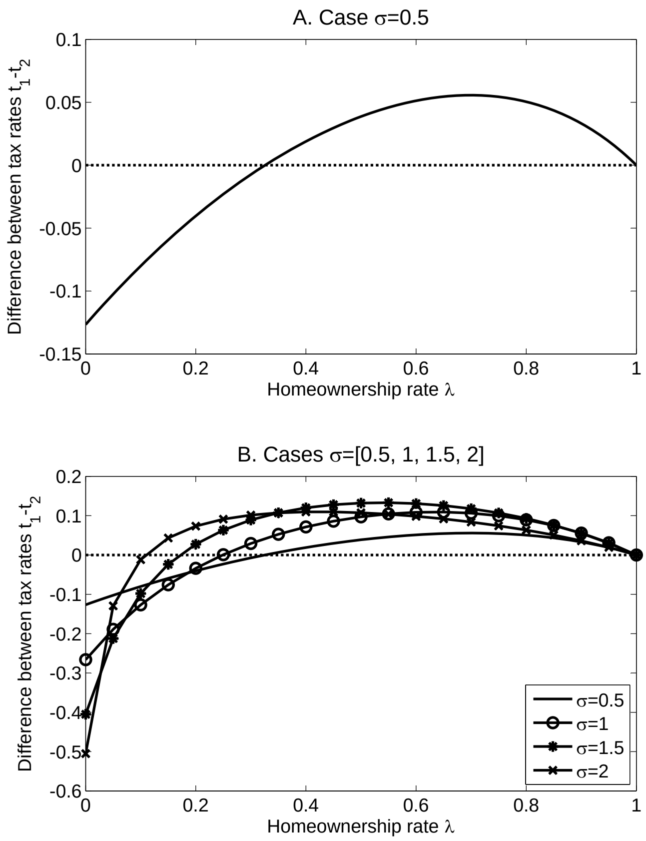


**Figure B.5:** Relationship Between the Degree of Mobility and Differences in Tax Rates

If there is no sorting equilibrium, a natural question to raise is what happens instead? From a theoretical standpoint it is clear: There is simply no stable equilibrium in the metropolitan area. Empirically, we would observe migration that proceeds uninterruptedly and frequent changes in tax rates.

### B.5.5 The Role of Initial Inequality

The parameter $\sigma$ which we interpret as a parameter characterizing the amount of inequality in the initial income distribution among all individuals (renters and homeowners) plays a crucial role for the threshold level $\tilde{\lambda}$. Consider the four cases for $\sigma$ in Panel B of Figure B.5. The higher $\sigma$ the bigger is the range of possible values for the homeownership rate from which on a sorting equilibrium arises. A numerical approximation of the threshold $\tilde{\lambda}$, implicitely defined in Proposition 2, as a function of $\sigma$ confirms this finding as shown in Figure B.6.[14]

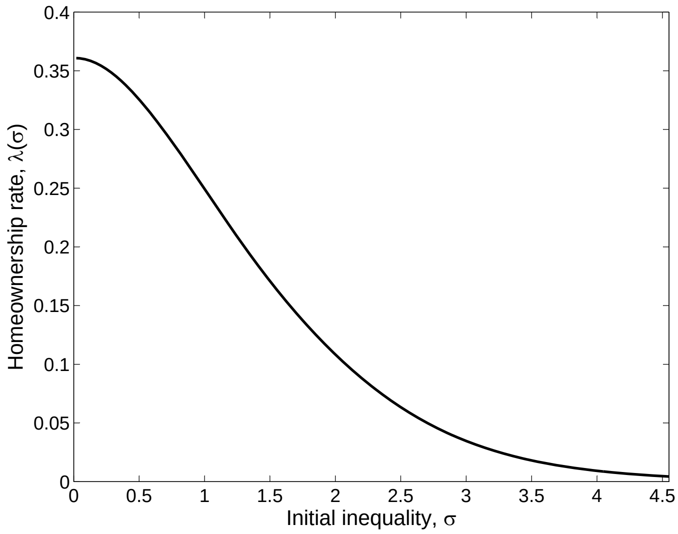


**Figure B.6:** Relationship Between Threshold and Initial Inequality

In the figure we can see that as the initial inequality $\sigma$ increases the threshold $\tilde{\lambda}$ becomes smaller, that is, even a small homeownership rate is already sufficient to lead to a sorting equilibrium, if the initial inequality in income is high. This can also be shown formally as summarized by the following Proposition:

[14]The numerical approximation is done using a cubic-spline-approximation method. For more details on function approximation methods see Judd (1998) or Miranda and Fackler (2002).

**PROPOSITION B.3.** *Suppose income is distributed lognormally across individuals. If $\sigma \to \infty$, there is always a sorting equilibrium independent of the homeownership rate in the economy.*

(The proof can be found in the Appendix). Because of the nature of the lognormal distribution a higher $\sigma$ increases the skewness of the income distribution.[15] The reason why there is no sorting equilibrium in some cases (especially for low homeownership rates) is that m-to-m inequality is too high among the rich part of the population compared to the poor part. In the range of low homeownership rates more homeownership then increases m-to-m inequality (after migration) among the poor more than among the rich, so that the higher the homeownership rate the more likely is a sorting equilibrium. The effect of a high initial inequality, $\sigma$, is now to exacerbate this effect of the homeownership rate. The higher $\sigma$ the less homeowners are needed to ensure that m-to-m inequality between the rich will be lower than between the poor.

## B.6 Conclusion

In this paper, we analyzed local redistribution in a political economy model where local governments have tax-grant instruments, part of the population is mobile (renters) and the other part immobile (homeowners). Voters are aware of migration effects of taxes and grants. This setting allows us to focus explicitly on the relationship between homeownership, mobility, income inequality and redistribution. In summary, our theoretical model leads to three predictions: (1) When mobility is high (low homeownership rate), income sorting does not arise as rich taxpayers in a certain community are too unequal between each other. (2) When part of the population is immobile (homeowners), income sorting may arise as part of the rich homeowners are

[15]For the lognormal distribution the mean is $\overline{y} = e^{\mu + \frac{\sigma^2}{2}}$ for a parameters $\mu$ and $\sigma$. Note, the higher $\sigma$ the less similar the lognormal distribution becomes to the normal distribution.

exploited by the poor renters while the rich renters may escape taxes. For high homeownership rates sorting is less likely as only few renters move and their influence on inequality is not sufficiently high. (3) Finally, when initial inequality in the economy is high even low homeownership rates can already induce income sorting.

Our work aims at contributing to the question, if redistributive policies can or should be taken out at the local level. On the one hand our results could be interpreted as showing that in the presence of homeownership there cannot be perfect income sorting, or in other words imperfect income sorting arises. A redistributive policy on the local level can, thus, be more effective compared to a world where income sorting is perfect. Another interpretation of our results in terms of redistributive policies could be that local income redistribution in a federal system is more effective if (initial) income inequality in the federation is not too high and homeownership is not at an intermediate degree.

Our paper is, however, just a first step in trying to investigate the relationship between homeownership, mobility and fiscal policy on the local level. The model presented here could be usefully extended by endogenizing the decision of homeownership. Another, potentially interesting extension to our current model would be to investigate a setting where mobility costs are less extreme, including, for example, some positive nonzero but finite mobility costs for some households. Furthermore, one could consider the influence of different assumptions about the relationship between mobility costs and income (or wealth) for the resulting income sorting equilibrium. For now this is left to future research.

## B.7 Appendix to the Paper

### B.7.1 Proof of Lemma 2

#### Distribution Functions

In jurisdiction 1, the low tax jurisdiction, only homeowners have an income below the boundary, so that for $y \leq \tilde{y}$ we have:

$$G_1(y) = \lambda F(y)$$

There are two kinds of people with income above the boundary: (1) homeowners with distribution $\lambda F(y)$ and (2) renters with distribution $(1-\lambda)\frac{F(y)-F(\tilde{y})}{1-F(\tilde{y})}$ (the fraction of people that have income between $\tilde{y}$ and $y$ divided by the total fraction of people with income above $\tilde{y}$). Thus we have the following distribution for people with income $y > \tilde{y}$:

$$\begin{aligned} G_1(y) &= \lambda F(y) + (1-\lambda)\frac{F(y) - F(\tilde{y})}{1 - F(\tilde{y})} \\ &= (2-\lambda)F(y) - (1-\lambda). \end{aligned}$$

as $x^M = 1 - F(\tilde{y}) = \frac{1}{2}$. The distribution of the population in jurisdiction 1 over income will thus have two different parts:

$$G_1(y) = \begin{cases} \lambda F(y) & y \leq \tilde{y} \\ (2-\lambda)F(y) - (1-\lambda) & y > \tilde{y} \end{cases}$$

Similarly, one can show for jurisdiction 2, the high tax jurisdiction:

$$G_2(y) = \begin{cases} (2-\lambda)F(y) & y \leq \tilde{y} \\ (1-\lambda) + \lambda F(y) & y > \tilde{y} \end{cases}.$$

#### Mean Incomes

In the low tax jurisdiction, jurisdiction 1, all homeowners have an mean income given by the original mean income, $\int_0^{\max y} y f(y) dy$ and all renters have a higher mean income given by $\int_{\tilde{y}}^{\max y} y \frac{f(y)}{1-F(\tilde{y})} dy$ (where the density is weighted by the total fraction of individuals with income higher than the $\tilde{y}$ to obtain the expectation of $y$ conditional on having an income higher than $\tilde{y}$ and the support is from the boundary to the maximum income). Thus, mean income in jurisdiction 1 is

$$\begin{aligned} \bar{y}_1 &= \lambda \int_0^{\max y} y f(y) dy + (1-\lambda) \int_{\tilde{y}}^{\max y} y \frac{f(y)}{1-F(\tilde{y})} dy \\ &= \lambda \int_0^{\max y} y f(y) dy + 2(1-\lambda) \int_{\tilde{y}}^{\max y} y f(y) dy, \end{aligned}$$

and similarly for jurisdiction 2:

$$\begin{aligned} \bar{y}_2 &= \lambda \int_0^{\max y} y f(y) dy + (1-\lambda) \int_0^{\tilde{y}} y \frac{f(y)}{F(\tilde{y})} dy \\ &= \lambda \int_0^{\max y} y f(y) dy + 2(1-\lambda) \int_0^{\tilde{y}} y f(y) dy. \end{aligned}$$

**Medians**

Principally there are four different cases for the relationship between median income and the income of the boundary mobile individuum:

1. $y^m \geq \tilde{y}$ in both communities,
2. $y_1^m \geq \tilde{y}$ and $y_2^m \leq \tilde{y}$,
3. $y_1^m \leq \tilde{y}$ and $y_2^m \geq \tilde{y}$,
4. $y^m \leq \tilde{y}$ in both communities.

We want to show that we must be in case 2 and that the medians are given by equations (B.5) and (B.6). To do this, we will proceed in the following order. First, we will show that if we are in case 2 the medians must be equal to (B.5) and (B.6). Then, we will show that given those medians we must be in case 2 for all relevant $\lambda$. Thus equations (B.5) and (B.6) represent the only relevant case for the medians.

**Part 1** Suppose $y_1^m \geq \tilde{y}$ and $y_2^m \leq \tilde{y}$. Then the median in jurisdiction 1 is implicitely defined as:

$$\begin{aligned} G_1(y_1^m) &= (2-\lambda)F(y_1^m) - (1-\lambda) = \frac{1}{2} \\ y_1^m &= F^{-1}\left(\frac{1}{2}\frac{3-2\lambda}{2-\lambda}\right). \end{aligned}$$

Similarly for the median in jurisdiction 2 we then have:

$$\begin{aligned} G_2(y_2^m) &= (2-\lambda)F(y_2^m) = \frac{1}{2} \\ y_2^m &= F^{-1}\left(\frac{1}{2}\frac{1}{2-\lambda}\right) \end{aligned}$$

Those are the medians given in equation (B.5) and (B.6)

**Part 2** Suppose the medians are given by $y_1^m = F^{-1}\left(\frac{1}{2}\frac{3-2\lambda}{2-\lambda}\right)$ and $y_2^m = F^{-1}\left(\frac{1}{2}\frac{1}{2-\lambda}\right)$. This implies

$$y_1^m = F^{-1}\left(\frac{1}{2}\frac{3-2\lambda}{2-\lambda}\right) \geq \tilde{y} = F^{-1}\left(\frac{1}{2}\right) \text{ for all } \lambda \leq 1$$

as $\frac{3-2\lambda}{2-\lambda} \geq 1$ for all $\lambda \leq 1$ and $F^{-1}(.)$ is a monotonously increasing function. Similarly:

$$y_2^m = F^{-1}\left(\frac{1}{2}\frac{1}{2-\lambda}\right) \leq \tilde{y} = F^{-1}\left(\frac{1}{2}\right) \text{ for all } \lambda \leq 1$$

as $\frac{1}{2-\lambda} \leq 1$ for all $\lambda \leq 1$ and $F^{-1}(.)$ is a monotonously increasing function. ■

## B.7.2 Derivation of Results of Example 1

Using Lemma 1 and and the distribution function given in the example we obtain for $\lambda = 0$:

$$
\begin{aligned}
y_1^m &= F^{-1}\left(\frac{3}{4}\right) = 1 + \frac{2}{3} = \frac{5}{3},\ \bar{y}_1 = 2\int_1^2 \frac{3}{8} y dy + 2\int_2^3 \frac{1}{8} y dy = \frac{7}{4} \\
&\Rightarrow\ t_1^* = \frac{1}{21} \\
y_2^m &= F^{-1}\left(\frac{1}{4}\right) = \frac{1}{2},\ \bar{y}_2 = 2\int_0^1 \frac{1}{2} y dy = \frac{1}{2} \\
&\Rightarrow\ t_2^* = 0
\end{aligned}
$$

and for $\lambda = \frac{1}{3}$:

$$
\begin{aligned}
y_1^m &= F^{-1}\left(\frac{7}{10}\right) = 1 + \frac{4}{9} = \frac{23}{15},\ \bar{y}_1 = \frac{1}{3}\overline{y} + \frac{4}{3}\left[\int_1^2 \frac{3}{8} y dy + \int_2^3 \frac{1}{8} y dy\right] = \frac{37}{24} \\
&\Rightarrow\ t_1^* = \frac{1}{185} \\
y_2^m &= F^{-1}\left(\frac{3}{10}\right) = \frac{3}{5},\ \bar{y}_2 = \frac{1}{3}\overline{y} + \frac{4}{3}\int_0^1 \frac{1}{2} y dy = \frac{17}{24} \\
&\Rightarrow\ t_2^* = \frac{13}{85}
\end{aligned}
$$

## B.7.3 Proof of Proposition 2

An income sorting equilibrium exists, if and only if $t_1^*(\lambda) < t_2^*(\lambda)$, or in words: the tax rate in the rich jurisdiction is lower than in the poor jurisdiction. We will show existence of an income sorting equilibrium for the case of $\lambda \geq \tilde{\lambda}$ in two steps. First, we will show that if $t_1^*(\lambda) < t_2^*(\lambda)$ which is the case for $\lambda \geq \tilde{\lambda}$ by definition of the threshold, then income sorting arises (Part 1). Second, we will show that if renters are sorted into jurisdictions according to income classes, we must have $t_1^*(\lambda) < t_2^*(\lambda)$ for all $\lambda \geq \tilde{\lambda}$ (Part 2).

**Part 1** Suppose we have $\lambda \geq \tilde{\lambda}$ and thus $t_1^*(\lambda) < t_2^*(\lambda)$. Note that we must have a lower grant-rent differential in jurisdiction 1, i.e. $(t_1^*(\lambda), g_1^*(\lambda) -$

$r_1^*(\lambda)) < (t_2^*(\lambda), g_2^*(\lambda) - r_2^*(\lambda))$. Otherwise, if the grant-rent differential was bigger in jurisdiction 1, everyone would like to live in jurisdiction 1. As all the jurisdictions must be populated and there is no extra space in jurisdiction 1. There would be an over-demand for living in jurisdiction 1 and the price of housing would rise there until the grant-rent differential becomes smaller than in jurisdiction 2 and some individuals agree to live in jurisdiction 2. What can be inferred about the distribution of renters over income in each jurisdiction? The ones that prefer a higher grant-rent to a lower tax rate will move to jurisdiction 2 while individuals preferring a lower tax rate to a higher grant-rent differential will move to jurisdiction 1. Totally differentiating shows that the slope of an indifference curve spanned by $t$ and $g - r$ is positive and increasing in income, i.e.

$$\left.\frac{d(g_j - r_j)}{dt_i}\right|_{V=\overline{V}} = y > 0.$$

Thus, low-income individuals prefer regions with higher taxes combined with large grant-rent differentials, whereas high-income individuals prefer regions with low taxes and high grant-rent differentials. Consequently, from the fact that not everyone can live in jurisdiction 1, $(t_1^*(\lambda), g_1^*(\lambda) - r_1^*(\lambda)) < (t_2^*(\lambda), g_2^*(\lambda) - r_2^*(\lambda))$ and that the relative preference for lower taxes versus a higher grant-rent differential depends on income, we can conclude that a sorting equilibrium must arise. Renters up to a certain boundary income $\tilde{y}$ live in jurisdiction 2 and individuals with higher income than the boundary $\tilde{y}$ live in jurisdiction 1.

**Part 2** Suppose there is income sorting. Then one can show that there is a threshold, $\tilde{\lambda}$, from which onwards an income sorting equilibrium exists. We procede in several steps. First, we derive the equation that defines the threshold (equation (B.7)) and then we show that the threshold really exists and is unique. For the lognormal distribution we have $F(y) = \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\ln(y)-\mu}{\sigma\sqrt{2}}\right)$ by definition with an overall mean given by $\bar{y} = \exp\left(\mu + \frac{\sigma^2}{2}\right)$ and an overall median given by $y^m = \exp(\mu)$. We now calculate the means and the medians for this distribution given income sorting.

**Means**

We can write

$$\int_{y^m}^{\infty} yf(y)dy = \exp\left(\mu + \frac{\sigma^2}{2}\right)\Phi(\sigma)$$

and

$$\int_{0}^{y^m} yf(y)dy = \exp\left(\mu + \frac{\sigma^2}{2}\right)(1 - \Phi(\sigma))$$

where $\Phi$ is cumulative distribution function of the standard normal. Thus equations (B.3) and (B.4) become

$$\begin{aligned} y_1 &= \lambda \exp\left(\mu + \frac{\sigma^2}{2}\right) + 2(1-\lambda)\exp\left(\mu + \frac{\sigma^2}{2}\right)\Phi(\sigma) \\ &= \exp\left(\mu + \frac{\sigma^2}{2}\right)[\lambda + 2(1-\lambda)\Phi(\sigma)] \end{aligned}$$

and

$$y_2 = \exp\left(\mu + \frac{\sigma^2}{2}\right)[\lambda + 2(1-\lambda)(1 - \Phi(\sigma))].$$

**Medians**

For the log normal distribution we obtain for $y_1^m$:

$$\begin{aligned} F(y_1^m) &= \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\ln(y_1^m) - \mu}{\sigma\sqrt{2}}\right) = \frac{1}{2}\frac{3-2\lambda}{2-\lambda} \\ y_1^m &= \exp\left(\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right) + \mu\right) \end{aligned}$$

and for $y_2^m$:

$$\begin{aligned} F(y_2^m) &= \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\ln(y_2^m) - \mu}{\sigma\sqrt{2}}\right) = \frac{1}{2}\frac{1}{2-\lambda} \\ y_2^m &= \exp\left(-\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right) + \mu\right) \end{aligned}$$

### Difference Between Tax Rates

Using the means and medians, we can now calculate the difference between tax rates

$$\begin{aligned} t_2 - t_1 &= 1 - \frac{y_2^m}{y_2} - 1 + \frac{y_1^m}{y_1} = \frac{y_1^m}{y_1} - \frac{y_2^m}{y_2} \\ &= \frac{\exp\left(\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right) + \mu\right)}{\exp\left(\mu + \frac{\sigma^2}{2}\right)\left[\lambda + 2(1-\lambda)\Phi(\sigma)\right]} \\ &\quad - \frac{\exp\left(-\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right) + \mu\right)}{\exp\left(\mu + \frac{\sigma^2}{2}\right)\left[\lambda + 2(1-\lambda)\left(1 - \Phi(\sigma)\right)\right]} \\ &= \frac{1}{\exp\left(\frac{\sigma^2}{2}\right)}\left(\frac{\exp\left(\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\right)}{\left[\lambda + 2(1-\lambda)\Phi(\sigma)\right]} - \frac{\exp\left(-\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\right)}{\left[\lambda + 2(1-\lambda)\left(1 - \Phi(\sigma)\right)\right]}\right). \end{aligned}$$

This shows (B.7).

### Existence and Uniqueness of Sorting Threshold

We would like to show formally that there exists a unique sorting threshold for the case of two communities with equal homeownership rates. We proceed in four steps:

**a** Show that $t_2 - t_1 = 0$ at $\lambda = 1$ for all $\sigma > 0$.

**b** Show that $t_2 - t_1 < 0$ at $\lambda = 0$ for all $\sigma > 0$.

**c** Show that there must be at least one $\lambda$ for which $t_2 - t_1 = 0$ in the interval $\lambda \in [0, 1[$.

**d** Show that there at most one $\lambda$ for which $t_2 - t_1 = 0$ in the interval $\lambda \in [0, 1[$.

**Part a** First, one can easily show that for any $\sigma$ the tax differential (B.7) is equal to zero at $\lambda = 1$:

$$
\begin{aligned}
t_2 - t_1 &= \frac{1}{\exp\left(\frac{\sigma^2}{2}\right)} \\
&\quad \left(\frac{\exp\left(\sigma\sqrt{2}\mathrm{erf}^{-1}(0)\right)\cdot 1 - \exp\left(-\sigma\sqrt{2}\mathrm{erf}^{-1}(0)\right)}{1}\right) \\
&= 0
\end{aligned}
$$

**Part b** At $\lambda = 0$ we will have for (B.7)

$$
\begin{aligned}
t_2 - t_1 &= \frac{1}{\exp\left(\frac{\sigma^2}{2}\right)} \\
&\quad \left(\frac{\exp\left(\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1}{2}\right)\right) 2\left(1-\Phi(\sigma)\right) - \exp\left(-\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1}{2}\right)\right) 2\Phi(\sigma)}{4\left(1-\Phi(\sigma)\right)\Phi(\sigma)}\right)
\end{aligned}
$$

To see that this must be below zero for all $\sigma$ consider the numerator (substituting constant terms with $c = \sqrt{2}\mathrm{erf}^{-1}\left(\frac{1}{2}\right) > 0$ for simplicity)

$$
\begin{aligned}
& \exp(c\sigma)\left(1-\Phi(\sigma)\right) - \exp(-c\sigma)\Phi(\sigma) \\
=\ & \exp(c\sigma)\left(1-\Phi(\sigma)\right) - \exp(-c\sigma)\left(1-\Phi(-\sigma)\right) \\
=\ & H(\sigma) - H(-\sigma)
\end{aligned}
$$

This shows that the numerator is the difference of a function $H(x) = \exp(cx)$ $(1-\Phi(x))$ at some point above zero with itself at some point below zero. Clearly if this function is decreasing this difference must be negative. The derivative of this function is given by

$$
\begin{aligned}
H'(x) &= c\exp(cx)(1-\Phi(x)) + \exp(cx)(-\phi(x)) \\
&= \exp(cx)\left[c(1-\Phi(x)) - \phi(x)\right]
\end{aligned}
$$

This derivative is negative because (1) the exponential term is positive and (2) the term in brackets is always negative. To see this consider

$$c(1-\Phi(x)) - \phi(x) < 0$$
$$c = \sqrt{2}\mathrm{erf}^{-1}\left(\tfrac{1}{2}\right) (\approx 0.67449) < \min\left[\tfrac{\phi(x)}{(1-\Phi(x))}\right] (\approx 0.79789) \quad \leq \frac{\phi(x)}{(1-\Phi(x))}.$$

**Part c** Consider the first derivative of the tax differential with respect to the homeownership rate:

$$\begin{aligned}\frac{\partial(t_2-t_1)}{\partial\lambda} &= \frac{1}{\exp\left(\frac{\sigma^2}{2}\right)}\left\{\frac{\exp(k_1 q(\lambda))\left[k_1 q'(\lambda)v_1(\lambda) - v_1'(\lambda)\right]}{v_1(\lambda)^2}\right. \\ &\quad \left. -\frac{\exp(k_2 q(\lambda))\left[k_2 q'(\lambda)v_2(\lambda) - v_2'(\lambda)\right]}{v_2(\lambda)^2}\right\}\end{aligned}$$

where we defined

$$\begin{aligned} k_1 &= -k_2 = \sigma\sqrt{2} \\ q(\lambda) &= \mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right) \\ q'(\lambda) &= \frac{\sqrt{\pi}}{2}\exp\left(q(\lambda)^2\right)\left(-\frac{1}{(2-\lambda)^2}\right) \\ v_1(\lambda) &= (1-2\Phi(\sigma))\lambda + 2\Phi(\sigma) \\ v_1'(\lambda) &= (1-2\Phi(\sigma)) \\ v_2(\lambda) &= (1-2\Phi(-\sigma))\lambda + 2\Phi(-\sigma) \\ v_2'(\lambda) &= (1-2\Phi(-\sigma)). \end{aligned}$$

At $\lambda = 1$ this first derivative is equal to

$$\begin{aligned}
& \frac{1}{\exp\left(\frac{\sigma^2}{2}\right)} \left\{ \frac{\exp(0)\left[\sigma\sqrt{2}\left(-\frac{\sqrt{\pi}}{2}\right) - (1-2\Phi(\sigma))\right]}{1} \right. \\
& \left. - \frac{\exp(0)\left[-\sigma\sqrt{2}\left(-\frac{\sqrt{\pi}}{2}\right) - (1-2\Phi(-\sigma))\right]}{1} \right\} \\
= \; & \frac{1}{\exp\left(\frac{\sigma^2}{2}\right)} \left\{ \sigma\sqrt{2}\left(-\frac{\sqrt{\pi}}{2}\right) - (1-2\Phi(\sigma)) - \sigma\sqrt{2}\left(\frac{\sqrt{\pi}}{2}\right) \right. \\
& \left. +(1-2\Phi(-\sigma)) \right\} \\
= \; & \frac{1}{\exp\left(\frac{\sigma^2}{2}\right)} \left\{ -\sigma\sqrt{2\pi} + 2(\Phi(\sigma) - \Phi(-\sigma)) \right\} < 0
\end{aligned}$$

as

$$\frac{2(\Phi(\sigma) - \Phi(-\sigma))}{\sigma} \leq \max\left(\frac{2(\Phi(\sigma) - \Phi(-\sigma))}{\sigma}\right)(\approx 1.6) < \sqrt{2\pi}(\approx 2.5).$$

This means that there is at least one intersection with zero in the interval $[0,1[$, because the tax differential function intersects at the point $\lambda = 1$ from above. From part 1 we know that at $\lambda = 1$ the tax difference $t_2 - t_1 = 0$ and from part 2 that at $\lambda = 0$ the tax difference is $t_2 - t_1 < 0$. This also implies that the number of intersections in the interval $[0,1[$ has to be odd.

**Part d** To see that there is a unique intersection expand (B.7) and set the numerator to zero

$$\begin{aligned}
& \exp\left(\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\right)\left[\lambda + 2(1-\lambda)\left(1-\Phi(\sigma)\right)\right] \\
& - \exp\left(-\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\right)\left[\lambda + 2(1-\lambda)\Phi(\sigma)\right] = 0 \\
& \exp\left(-\sigma 2\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\right) - \frac{1-(1-\lambda)\mathrm{erf}\left(\frac{\sigma}{\sqrt{2}}\right)}{1+(1-\lambda)\mathrm{erf}\left(\frac{\sigma}{\sqrt{2}}\right)} = 0 \\
& m(\lambda) - n(\lambda) = 0
\end{aligned}$$

Note that we have already shown that $n(0) > m(0)$ for any $\sigma$. If we can show that $n(\lambda)$ and $m(\lambda)$ are both convex increasing functions, then it is clear that they have at most two intersections. See, Figure B.7 as an illustration for $\sigma = 1$.

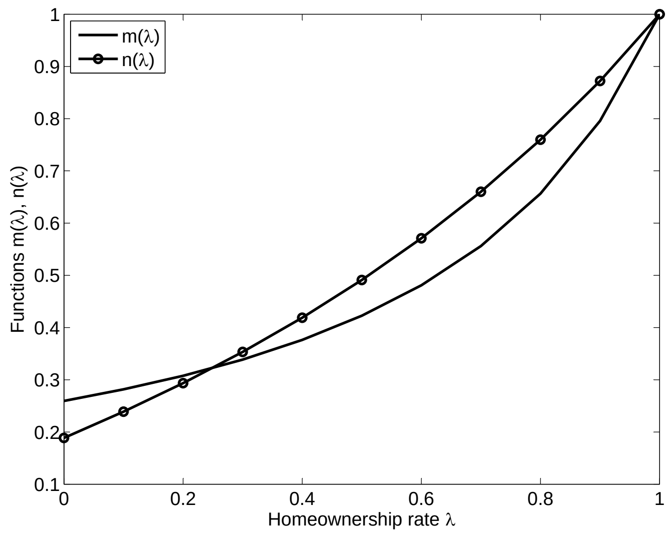


**Figure B.7:** Proof of Unique Sorting Threshold: An Illustration

Together with part 3 of the proof, where we have shown that there must be at least one threshold, this would complete the proof that the threshold is unique.

It is easy to show that the function $n(\lambda)$ is a convex increasing function:

$$\begin{aligned}
\frac{\partial n(\lambda)}{\partial \lambda} &= \frac{2\text{erf}\left(\frac{\sigma}{\sqrt{2}}\right)}{\left[1-(1-\lambda)\text{erf}\left(\frac{\sigma}{\sqrt{2}}\right)\right]^2} > 0 \text{ as erf}(x) > 0 \text{ for } x > 0 \\
\frac{\partial^2 n(\lambda)}{(\partial \lambda)^2} &= \frac{4\text{erf}\left(\frac{\sigma}{\sqrt{2}}\right)}{\left[1-(1-\lambda)\text{erf}\left(\frac{\sigma}{\sqrt{2}}\right)\right]^3} > 0 \text{ as erf}(x) > 0 \text{ for } x > 0,\ (1-\lambda) < 1 \\
&\quad \text{anderf}\left(\frac{\sigma}{\sqrt{2}}\right) < 1
\end{aligned}$$

The function $m(\lambda)$ is also clearly an increasing function:

$$\frac{\partial m(\lambda)}{\partial \lambda} = \frac{\exp\left(\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\left(-2\sqrt{2}\sigma + \mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\right)\right)\sqrt{2\pi}\sigma}{(2-\lambda)^2} > 0$$

But at a first glance it is not clearly convex or concave:

$$\frac{\partial^2 m(\lambda)}{(\partial \lambda)^2} = \frac{1}{(\lambda-2)^4}\exp\left(\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\left(2\sqrt{2}\sigma + \mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\right)\right)\sqrt{2\pi}\sigma$$
$$\left[4 + \exp\left(\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)^2\right)\sqrt{2\pi}\sigma - 2\lambda - \exp\left(\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)^2\right)\right.$$
$$\left.\sqrt{\pi}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\right]$$

The term in square brackets determines, if this second derivative is positive or negative, because all other terms are positive.

As a preliminary consider that on the interval $\lambda \in [0,1[$ we have:

$$\begin{aligned} 0 &\leq \mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right) < \frac{1}{2} \\ 0 &\leq \mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)^2 < \frac{1}{4} \end{aligned}$$

Now we want to show that the square brackets are positive. We split the square brackets into two terms $p_1(\lambda)$, the positive part, and $-p_2(\lambda)$, the negative part:

$$\begin{aligned} p_1(\lambda) &= 4 + \exp\left(\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)^2\right)\sqrt{2\pi}\sigma \\ p_2(\lambda) &= 2\lambda + \exp\left(\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)^2\right)\sqrt{\pi}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right) \end{aligned}$$

In the following, we will show that $p_1(\lambda) > p_2(\lambda)$, which means that the term in square brackets must be positive. Instead of comparing $p_1(\lambda)$ and $p_2(\lambda)$

we can also compare the lower bound of $p_1(\lambda)$ with respect to $\lambda$ on the given interval: $\min_\lambda [p_1(\lambda)] \leq p_1(\lambda)$ for all $\lambda \in [0, 1[$ with an upper bound of $p_2(\lambda)$ with respect to $\lambda$ on the given interval: $\max_\lambda [p_2(\lambda)] \geq p_2(\lambda)$ for all $\lambda \in [0, 1[$. Those boundaries are given by:

$$\begin{aligned}\min_\lambda [p_1(\lambda)] &= 4 + \sqrt{2\pi}\sigma \\ \max_\lambda [p_2(\lambda)] &= 2 + \exp\left(\frac{1}{4}\right)\frac{1}{2}\sqrt{\pi}\end{aligned}$$

Comparing them we see that

$$4 + \sqrt{2\pi}\sigma > 2 + \exp\left(\frac{1}{4}\right)\frac{1}{2}\sqrt{\pi}(\approx 3.1379) \text{ for all } \sigma \geq 0$$

which completes the proof. ■

### B.7.4 Proof of Proposition 3

Consider the difference between tax rates:

$$t_2 - t_1 = \frac{1}{\exp\left(\frac{\sigma^2}{2}\right)}\left(\frac{\exp\left(\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\right)}{[\lambda + 2(1-\lambda)\Phi(\sigma)]} - \frac{\exp\left(-\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)\right)}{[\lambda + 2(1-\lambda)(1-\Phi(\sigma))]}\right)$$

We want to show that:

$$\lim_{\sigma\to\infty}(t_2 - t_1) = 0$$

Rewrite the tax difference as follows:

$$t_2 - t_1 = \frac{\exp\left(\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right) - \frac{\sigma^2}{2}\right)}{[\lambda + 2(1-\lambda)\Phi(\sigma)]} - \frac{\exp\left(-\sigma\sqrt{2}\mathrm{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right) - \frac{\sigma^2}{2}\right)}{[\lambda + 2(1-\lambda)(1-\Phi(\sigma))]}$$

First note that the denominators' limiting values are constant:

$$\begin{aligned}\lim_{\sigma\to\infty}[\lambda + 2(1-\lambda)\Phi(\sigma)] &= 2 - \lambda \\ \lim_{\sigma\to\infty}[\lambda + 2(1-\lambda)(1-\Phi(\sigma))] &= \lambda\end{aligned}$$

The second term thus goes to zero:

$$\lim_{\sigma\to\infty} \frac{\exp\left(-\sigma\sqrt{2}\text{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)-\frac{\sigma^2}{2}\right)}{\left[\lambda+2(1-\lambda)\left(1-\Phi(\sigma)\right)\right]} = \frac{\exp(-\infty)}{\lambda} = 0$$

The first term also goes to zero because the square term dominates:

$$\lim_{\sigma\to\infty} \frac{\exp\left(\sigma\sqrt{2}\text{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)-\frac{\sigma^2}{2}\right)}{\left[\lambda+2(1-\lambda)\Phi(\sigma)\right]} = \frac{\exp(-\infty)}{2-\lambda} = 0$$

Thus we must have:

$$\begin{aligned}
\lim_{\sigma\to\infty}(t_2-t_1) &= \lim_{\sigma\to\infty} \frac{\exp\left(\sigma\sqrt{2}\text{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)-\frac{\sigma^2}{2}\right)}{\left[\lambda+2(1-\lambda)\Phi(\sigma)\right]} \\
&\quad - \lim_{\sigma\to\infty} \frac{\exp\left(-\sigma\sqrt{2}\text{erf}^{-1}\left(\frac{1-\lambda}{2-\lambda}\right)-\frac{\sigma^2}{2}\right)}{\left[\lambda+2(1-\lambda)\left(1-\Phi(\sigma)\right)\right]} \\
&= 0
\end{aligned}$$

■

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# Appendix C

# Public Debt in a Political Economy


### Paper Summary

In this paper I analyze the determination of government debt in a dynamic politico-economic model with overlapping generations and a closed bond market. Government debt is on the one hand an investment instrument for young agents to provide old age support and on the other hand a strategic instrument for voters today to influence the decision of tomorrow's voters. In the closed economy where the resource constraint has to hold the level of government debt has no effect on today's total available resources. However it has an influence on the allocation of resources to public or private consumption. Under commitment government debt is solely determined by the tradeoff between public and private goods in old age. In the political equilibrium there is additionally a 'strategic effect' of government debt. By setting a higher level of government debt, today's generation of young voters can rip off tomorrow's generation of young voters by inciting a higher taxation. The more substitutable public and private goods are, the stronger is the strategic effect.

## C.1 Introduction

Since the end of the financial crisis of 2008/2009 Europe has fallen into a debt-crisis. Especially the governments of Greece, Ireland, Spain and Portugal are experiencing high risk premia on their bonds and troubles with their government finances. How can it happen that those relatively "rich" countries end up in such an unsustainable situation of high indebtedness? What characteristics of a country lead to high indebtedness in theory? Which role does the financial openness of a country play? And finally what institutional mechanisms are more prone to induce high government debt?

This paper aims at providing some answers to those questions. In line with the recent literature on the political economy of government debt, the inability of democratic governments to commit to policies over long periods of time is viewed as crucial for an understanding of how the level of government debt is determined (see for example Alesina and Tabellini, 1990; Persson and Svensson, 1989; Song et al., 2009; Debortoli and Nunes, 2010; Battaglini and Coate, 2008; Acemoglu et al., 2011). A government that is re-elected every period cannot directly decide on what fiscal policy is implemented in the future, because the next government will be in place by then and may change existing plans. As already noted by Persson and Svensson (1989) and Alesina and Tabellini (1990) governments that are re-elected every period may thus try to influence strategically decisions of the next period's government by setting a specific level of government debt.

More precisely, in this paper, I investigate the politico-economic motives behind the determination of public debt in a model with young and old voters. To this aim, I analyze an overlapping generations model where agents live for two periods. There is a first period when the agent works and a second period when he retires and lives of his savings. In both periods the agent consumes public goods in addition to his private consumption. I focus on Markov-perfect equilibria where government debt is the only payoff-relevant state variable. Government debt is on the one hand an investment instrument for young agents to provide old age support and on the other hand a strategic instrument for the government today to influence the decision of tomorrow's government. Because I assume a closed economy with an endogenous interest rate and the resource constraint has to hold, government debt can however not be used to finance expenditures today. In fact, there is an intertemporal dichotomy in the model, where previous government debt, taxes and public goods determine the equilibrium allocation today, whereas new government debt only matters for the equilibrium allocation tomorrow.

To gain intuition and as a benchmark for comparison, I first analyze the commitment solution, where the first generation decides on the whole future path of taxes, public goods and government debt. The old agents do not care about government debt at all, because it is not useful to finance public expenditures today. For the young agents the role of government debt issues is to achieve the optimal mix between private and public consumption in old age. The bond proceeds will be consumed privately by today's young, tomorrow in their old age. The consumption units promised in form of the bonds, however, have to be paid out by the government which restrains tomorrow's budget and thus constrains public good provision tomorrow. Thus, under commitment, the young agents today choose new government debt issues such that they optimally solve their tradeoff between private and public consumption tomorrow.

In the political equilibrium, there is additionally a 'strategic effect' on the decisions of future voters. Young voters today know that the reaction of the next generation of voters to a high level of government debt will be to reduce public expenditures and increase taxes to be able to finance the debt burden. They can thus gain from setting a higher level of government debt to induce a higher taxation of tomorrow's young generation. In this way, the young generation today can "rip off" the next generation of young. To achieve this the young today have to be ready to substitute public with private goods tomorrow. Thus the more substitutable public and private goods are the more they will make use of this strategic effect. Regarding the answer to the question which institutional mechanisms are particularly prone to favor a high debt accumulation, this paper thus shows that it depends on the substitutability of public and private goods. If public and private goods are substitutes, a commitment mechanism leads to a lower debt level than a voting mechanism. If, however, public and private goods are complements, a voting mechanism leads to a lower debt level than the commitment mechanism.

A comparative statics analysis yields an answer to the question what characteristics of a country can lead to high levels of government debt. The more concern there is for public goods provision or the more altruistic parents are in a country the lower will be the level of government debt. The influence of the political power of old voters on the level of government debt depends on whether public goods are substitutes or complements. When the power of the old voters increases the level of debt resembles more the one under commitment, i.e. under complementarity it will be higher and under substitutability it will be lower.

A closely related contribution is the paper by Song et al. (2009) who analyze a similar model for a small open economy with a constant interest rate. They show that under distortive taxation the fact that the young are concerned about public good provision in the next period prevents government debt to rise to the maximal sustainable level. One contribution of this paper is to compare the results to those of the small open economy model presented in Song et al. (2009). Such an analysis is important, because since the findings of Feldstein and Horioka (1980) it is well-known that savings and investments in a country are statistically correlated, which puts the small open economy assumption in its purest form, i.e. with a constant interest rate, into question. Therefore the reality is probably somewhere in between with an international financial market willing to absorb some of the bonds, but not an unlimited amount of them, so that to some extent the interest rate will react to bond issues. Interestingly, the analysis in this paper shows that endogenizing the interest rate leads to very different results for the politico-economic determination of government debt. Even without distortive taxation an interior debt level is reached which is lower than the maximum. This could be an explanation for the general increase in government debt levels in the developed economies during the last decades. Financial openness could be a factor to favor the building up of high government debt, as the government is not constrained to a national market but can sell the bonds on the international financial market with deeper pockets. However, a caveat is that, as shown in an extension, under distortive taxation this result can be reversed. In fact, when tax distortions are high enough government debt can even be lower in open economies. The reason is that if the costs of providing bonds are high (for example, because of efficiency losses arising from tax distortions), government debt will be reduced drastically in the open economy because of the concern of young voters for future public goods provision, as shown by Song et al. (2009). However, in the closed economy, some bonds are necessary as they provide the (only) saving instrument for the young agents. As a result, government debt is higher in the closed economy in this case. Thus the distortiveness of taxation is a crucial factor to consider to understand the influence of financial openness on the politico-economic determination of government debt.

This paper is complementary to recent work about the determination of public debt in models without commitment. Debortoli and Nunes (2010) analyze optimal fiscal policy in an economy where governments with different preferences alternate in office. Battaglini and Coate (2008) present a dynamic legislative bargaining model where legislators can distribute revenues back to their districts via pork-barrel spending. Acemoglu et al. (2011) study the dynamic taxation of capital and labor in the Ramsey model, but under the assumption that taxes and public goods are determined by a self-interested rent-seeking politician who cannot commit to future policies. Those authors all emphasize different channels which may lead to an overaccumulation of public debt. Here the focus is on the general-equilibrium effect and the political conflicts between different generations of voters. An important difference to previous work is that (similarly to Song et al., 2009) the inability of governments to commit does not necessarily lead to a higher level of government debt.

The paper is structured as follows. Section 2 presents the basic model. Section 3 discusses the implications of the model. Section 4 presents an extension with elastic labor supply and distortive taxation and discusses its implications. Section 5 concludes.

## C.2 A Simple Model with an Endogenous Interest Rate

The model presented here is based on Song et al. (2009), but with the important difference that the interest rate is endogenous. For simplicity agents live for two periods only. In the first period they work and in the second period they retire and live of their savings. The population size is constant. There is no production sector. The government can tax income, issue bonds, supply public goods and has to pay back government debt from previous periods.

### C.2.1 Private Maximization Problem

The individual has the choice between two kinds of consumption: private goods consumption ($c$) and public goods consumption ($g$). Agents are altruistic towards their children. The preferences of a young agent in period $t$ can be summarized by the following utility function:

$$U_{Y,t} = u(c_{Y,t}) + \theta u(g_t) + \beta \Big[ u(c_{O,t+1}) + \theta u(g_{t+1}) + \lambda U_{Y,t+1} \Big] \tag{C.1}$$

where $c_{Y,t}$ is private consumption of the agent in his youth in period $t$, $g_t$ is public consumption in his youth, $c_{O,t+1}$ is private consumption in his old age in period $t+1$, $g_{t+1}$ is public consumption in his old age, $\theta$ is the preference for public goods relative to private goods, $\beta$ is the time preference or patience and $\lambda$ is the degree of altruism. Note that $\lambda = 0$ nests a pure OLG model, where everyone only cares about his own utility and $\lambda = 1$ nests the case of perfect altruism or dynasties (infinitely lived agents).
Correspondingly an old agent in period $t$ has the following utility function:

$$U_{O,t} = u(c_{O,t}) + \theta u(g_t) + \lambda U_{Y,t} \tag{C.2}$$

I assume a CES-utility function in the following way:

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma} \tag{C.3}$$

Note that this specification implies that the elasticity of intertemporal substitution and the elasticity of substitution between public and private goods are both equal to $\frac{1}{\sigma}$. I also assume supplied hours of labor to be constant and equal to 1: $h = 1$, such that the gross income of an agent, $y_t$, is given by:

$$y_t = w$$

where $w$ is a constant wage parameter. Note that this assumption implies that there are no tax distortions and taxation is effectively lump sum. Income net of taxes is thus given by:

$$A(\tau_t) = (1 - \tau_t)w$$

Private agents can save for their retirement by buying one period bonds from their own national government. The budget constraints of the private agents are then given by:

$$\begin{aligned} c_{Y,t} + p_t b^d_{t+1} &= A(\tau_t) \\ c_{O,t+1} &= b^d_{t+1} \end{aligned}$$

where $p_t$ is the price of bond for bonds sold at time $t$ and $b^d_{t+1}$ represents the number of units of consumption promised at time $t$ delivered at time $t+1$ (demand for bonds). The gross interest rate will then be $R_{t+1} = \frac{1}{p_t}$.

The young have to decide how much to consume at time $t$ in their youth and at time $t+1$, when they are old. They take the price of bond, the tax rate and government consumption as given. Therefore the maximization problem of the consumer looks as follows:

$$\begin{aligned} &\max_{c_{Y,t}, c_{O,t+1}} u(c_{Y,t}) + \theta u(g_t) + \beta \Big[ u(c_{O,t+1}) + \theta u(g_{t+1}) + \lambda U_{Y,t+1} \Big] \\ &s.t. \quad c_{Y,t} + p_t b^d_{t+1} = A(\tau_t) \\ &\qquad c_{O,t+1} = b^d_{t+1} \end{aligned}$$

Solving this problem yields the typical Euler equation which defines the tradeoff between consumption today and tomorrow as a function of the price of bond:

$$p_t = \beta \frac{u'(c_{O,t+1})}{u'(c_{Y,t})}$$

The unique and interior solutions for the private consumption choices of old and young agents are given by:

$$c^*_{O,t+1}(\underset{-}{\tau_t}, \underset{-}{p_t}, \underset{+}{\beta}, \underset{+}{w}) = \frac{1}{p_t} \frac{\beta^{\frac{1}{\sigma}}}{p_t^{\frac{1-\sigma}{\sigma}} + \beta^{\frac{1}{\sigma}}} A(\tau_t) = b^d_{t+1} \tag{C.4}$$

$$c^*_{Y,t}(\underset{-}{\tau_t}, p_t, \underset{-}{\beta}, \underset{+}{w}) = \frac{p_t^{\frac{1-\sigma}{\sigma}}}{\left(p_t^{\frac{1-\sigma}{\sigma}} + \beta^{\frac{1}{\sigma}}\right)} A(\tau_t) \tag{C.5}$$

A higher wage income naturally has a positive effect on consumption for young and old and a higher tax rate correspondingly has a negative effect on both consumptions as it lowers net income of the agents. A higher discount factor has the effect of shifting consumption from today to tomorrow, because more patient agents like to save more. It is clear that a lower price of bond meaning a higher return on bonds increases consumption of the old. However note that the effect of the interest rate on consumption of the young depends on the elasticity of intertemporal substitution.

### C.2.2 Government Constraints

The government has to balance expenditures with revenues. The revenues consist of new bonds that are issued and tax revenues: $p_{t+1}b^s_{t+1} + \tau_t w$, where $b^s_{t+1}$ is the government supply of bonds, which pay one unit of consumption at time $t+1$. The expenditures consist of the debt to repay (in units of consumption) which is given from last periods and the government expenditure for provision of the public good: $g_t + b_t$. Therefore the budget constraint of the government looks as follows:

$$p_t b^s_{t+1} = g_t + b_t - \tau_t w$$

The government thus has to provide the promised units of time $t$ consumption ($b_t$) and it issues new bonds $b^s_{t+1}$. In the decision problem of the government, debt to repay is thus a liability from last period, which is taken as given. Therefore $b_t$ will be defined as the state variable in the decision problem of the government. Because $c_{O,t} = b_t$ I will denote the state variable interchangeably by $c_{O,t}$ or $b_t$.

When the bond market clears, $b^s_{t+1} = b^d_{t+1}$ and using the first order conditions the government budget constraint simplifies to:

$$g_t = w - c_{Y,t} - c_{O,t} \tag{C.6}$$

Note that government spending in $t$ is a function of the consumption of the young, $c_{Y,t}$, and the state variable, $c_{O,t}$, only. This implies that the new bonds that are issued, $b_{t+1}$, do not affect the budget of the government given the consumption allocation of the young. The reason is that the government cannot simply finance its expenditures by borrowing from abroad like in Song et al. (2009). The bonds have to be sold to the young agents today and those have to be willing to give up some consumption. Equation (C.6) is a resource constraint that the government has to respect.

In this paper I abstract from default and assume that the government is committed to pay back the debt. In the small open economy model (SOE) of Song et al. (2009) there is a natural debt limit given by the present discounted value of the maximum tax revenue that can be collected:[1]

$$\bar{b}_{\text{SOE}} = \frac{R\bar{\tau}w}{(R-1)}$$

where $R$ denotes the interest rate equivalent to the inverse bond price, $w$ is the endowment like here, and $\bar{\tau}$ denotes the maximal tax rate. The reason is that the government cannot go beyond this limit without repudiating debt at some future point in time. Thus this debt limit arises naturally from the assumption that the government is committed not to repudiate the debt. In case of an endogenous interest rate, this natural debt limit doesn't exist because the government influences the interest rate with fiscal policy.

The concept most closely resembling the one of a debt limit in this closed economy model is a feasibility limit on private consumption of the old next period. As we have seen, debt is equal to consumption of the old. There

[1] In the model by Song et al. (2009) government debt is defined as before interest, whereas in the model presented in this paper government debt is defined for reasons of technical simplicity as government debt after interest. Denote government debt in the model by Song et al. (2009) by $\tilde{b}$ and government debt in this model simply by $b$. To make the two concepts comparable one can redefine the debt in the model by Song et al. (2009) to correspond to $b$: $b = R \cdot \tilde{b}$. As the interest rate is constant in the small open economy this is simply a multiplication by a constant.

is a clearly defined "natural" upper bound for this variable given by the resource constraint. Consumption of the old cannot be larger than the available resources, $w$:

$$c_{O,t+1} = b_{t+1} < w \tag{C.7}$$

Thus there is a different "debt limit" in the closed economy model (CE) given by:

$$\bar{b}_{\mathrm{CE}} = w \tag{C.8}$$

### C.2.3 The Commitment Problem

As a first step, I analyze the case of commitment where the first generation of voters can fix future policies once and for all. More precisely, the solution to the commitment problem is defined as a feasible path of the policy variables (the tax rate, $\tau_t$, the public good, $g_t$, and the level of government debt, $b_{t+1}$) that would be chosen by the first generation, if they could decide on the whole path of each of the variables over time. To be able to find an analytical solution I reformulate the problem in terms of allocations using the bond market clearing condition, the budget constraints and the first order conditions of the agents. Equation (C.6) defines $g_t$ in terms of consumption allocations:

$$g_t = w - c_{Y,t} - c_{O,t}$$

The following system of equations yields $p_t$, $b_{t+1}$ and $\tau_t$ in terms of consumption allocations (where the bond market clearing is assumed to hold):

$$\begin{aligned} p_t &= \beta \frac{u'(c_{O,t+1})}{u'(c_{Y,t})} \\ b_{t+1} &= c_{O,t+1} \\ (1-\tau_t)w &= c_{Y,t} + p_t b_{t+1} \end{aligned}$$

When I have found the solution of the problem in terms of consumption allocations I can then recover the corresponding price of bond, tax rate and government bond issues using this system of equations. Such an approach is called "primal" and is frequently used in the optimal taxation literature.

To understand how the commitment problem can be formulated it is useful to first look at the case where only the old decide, then the case where only the young decide and finally at the more general formulation where both groups are assigned their corresponding weight.

**Only the Old Decide**

First suppose only the old agents decide. Then their optimization problem can be formulated in a recursive way with the following Bellman equation (see Appendix for more details):

$$\begin{aligned} &V_O^{Comm}(c_O) = \max_{c_Y, c'_O, g} \left\{ v(c_Y, c_O, g) + \lambda\beta V_O^{Comm}(c'_O) \right\} \\ &\text{s.t. } g = w - c_Y - c_O \end{aligned} \tag{C.9}$$

where $v(c_Y, c_O, g) = u(c_O) + (1+\lambda)\theta u(g) + \lambda u(c_Y)$. As already mentioned above I define consumption of the old today (which is equal to the inherited level of government debt from last period) as the state variable, which summarizes all past events. Note that consequently old agents today cannot decide on their consumption today, because this is already determined by their bond holdings (the savings decision in their youth). They decide on consumption of the young today and consumption of the old tomorrow. The first order conditions of this problem are given by (see Appendix for more details):

$$-\theta(1+\lambda)u'(g) + \lambda u'(c_Y) = 0 \tag{C.10}$$

$$\lambda\beta[u'(c'_O) - \theta(1+\lambda)u'(g')] = 0 \tag{C.11}$$

Equation (C.11) shows that consumption of the old, or equivalently government debt has to be constant, because $c_Y$ doesn't show up in the equation. In fact, in the closed economy model there is an intertemporal dichotomy in the problem. $c_Y$ determines the allocation of total resources ($w$) between private and public consumption today and $c'_O$ determines the allocation between private and public consumption tomorrow. Therefore the choice of $c'_O$ is only relevant for the future and does not depend on the state of the economy $c_O$. Intuitively as alluded above, government debt cannot be used as a financing instrument for government spending this period, because the government has to sell the bonds to the young agents inside the economy and cannot borrow funds from the international capital market.

Solving equations (C.10) and (C.11) for consumption of the young, $c_Y$, and consumption of the old next period, $c'_O$, under the assumption of CES-utility one obtains the following solutions (see Appendix for more details):

$$\begin{aligned} c_{Y,t} &= \begin{cases} \frac{\lambda^{\frac{1}{\sigma}}}{\left((\theta(1+\lambda))^{\frac{1}{\sigma}}+\lambda^{\frac{1}{\sigma}}\right)}(w - c_{O,t}) & \text{in } t = 0 \\ \frac{\lambda^{\frac{1}{\sigma}}}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}}+1+\lambda^{\frac{1}{\sigma}}\right]} w & \text{in } t > 0 \end{cases} \\ c_{O,t+1} &= b_{t+1} = \frac{1}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}}+1+\lambda^{\frac{1}{\sigma}}\right]} w \text{ for } \lambda, \beta > 0 \text{ in } t \geq 0 \end{aligned}$$

To understand better the determination of the debt level in this model consider the case of no altruism. If there was no altruism ($\lambda = 0$), the old would choose $c_Y = 0$ and $g = w - c_{O,0}$ to receive the maximum amount of public goods consumption for themselves. This means that they tax away the whole income of the young generation by setting the maximal tax rate $\tau = 1$. However if we look at the F.O.C. for consumption of the old in equation (C.11) we note that if $\lambda = 0$ the consumption of the old is indeterminate (or equivalently public debt is indeterminate). That is the old do not care about consumption of the next generation of old in this case. Clearly, as they care only about public goods consumption today it is sufficient for them to set consumption of the young today to a minimal level. Whether the young are promised any consumption in the next period to compensate them for the loss or not does not interest the old, because they will not be alive anymore.

In contrast, if there is altruism ($\lambda > 0$) the level of government debt is not indeterminate. The reason is that the old care about their children which in turn are affected by government debt in two ways: (1) the bonds they own constitute their consumption in old age and (2) the debt represents a burden for the government budget next period which means that less public goods can be provided. The level of debt therefore solves the tradeoff between public and private consumption of the old of tomorrow (the young of today). The old of today will consider this if they are altruistic.

### Only the Young Decide

This subsection analyzes the case where only the young agents decide. Their problem can be formulated recursively in two stages (see Appendix for more details):

$$\begin{aligned} \{c_{Y,0}, c_{O,1}, g_0\} &= \arg\max \left\{u(c_{Y,0}) + \theta u(g_0) + V_O^{Comm}(c_{O,1})\right\} \quad \text{(C.12)} \\ V_O^{Comm}(c_O) &= \max_{c_Y, c_O', g} \left\{v(c_Y, c_O, g) + \lambda\beta V_O^{Comm}(c_O')\right\} \text{ for } t > 0 \\ \text{s.t. } g &= w - c_Y - c_O \end{aligned}$$

where $v(.)$ is defined as above. As we can see, the commitment problem where only the young decide is different only in period 0. From period 1 onwards it is the same as if only the old decide (see equation (C.9)). Therefore it is not surprising that the solutions for $c_{Y,t}$ and $c_{O,t+1}$ are the same as in the case where only the old decide with the exception of $c_{Y,0}$ (see Appendix for more details):

$$c_{Y,t} = \begin{cases} \frac{1}{\left(1+\theta^{\frac{1}{\sigma}}\right)}(w - c_{O,t}) & \text{in } t = 0 \\ \frac{\lambda^{\frac{1}{\sigma}}}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}} + 1 + \lambda^{\frac{1}{\sigma}}\right]} w & \text{in } t > 0 \end{cases}$$

$$c_{O,t+1} = b_{t+1} = \frac{1}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}} + 1 + \lambda^{\frac{1}{\sigma}}\right]} w \quad \text{for } \lambda, \beta > 0 \text{ in } t \geq 0$$

Consider again the case of no altruism to gain intuition. In period 0 the young set their consumption such that resources at their disposition ($w - c_{O,0}$) are optimally split between public and private consumption. Private consumption is a share of $\frac{1}{1+\theta^{\frac{1}{\sigma}}}$ of those available resources and public consumption a share of $\frac{\theta^{\frac{1}{\sigma}}}{1+\theta^{\frac{1}{\sigma}}}$. Note that the higher the share of public consumption compared to private consumption the higher taxes have to be. Thus the young will tax themselves according to their preference for public goods, $\theta$, taking the state variable $c_{O,0}$ as given. Logically, the higher the preference for public goods, $\theta$, the higher the share of public consumption will be. Furthermore the higher the elasticity of substitution, $\frac{1}{\sigma}$, the higher the share of public consumption, if $\theta < 1$ and the lower the share of public consumption, if $\theta > 1$. Take the natural case of $\theta < 1$. In this case private consumption has a higher weight in the utility function. To the extent that different sorts of consumption are substitutable (measured by $\frac{1}{\sigma}$) they thus prefer to consume privately. A higher substitutability thus lets them substitute public by private consumption and the share of public consumption will be lower.

In period 1, the young set private consumption of their children to zero, $c_{Y,1} = 0$, as they are not altruistic. This means they tax away their whole income and use it for bond repayments (or equivalently their own private consumption in old age) and public goods provision. Their disposable income is thus equal to $w$. As intra-temporal utility is symmetric in old age and youth they will split disposable income in the same way as before between private and public consumption. To achieve this optimal mix, they set the number of bonds to $c_{O,1} = \frac{1}{1+\theta^{\frac{1}{\sigma}}} w$. Thus a share $\frac{1}{1+\theta^{\frac{1}{\sigma}}}$ of income is consumed privately and a share $\frac{\theta^{\frac{1}{\sigma}}}{1+\theta^{\frac{1}{\sigma}}}$ of income is consumed publicly. Logically, $c_{O,t+1}$ for $t > 1$ is again indeterminate in the absence of altruism.

### The General Commitment Solution

If young and old decide both together according to their weight ($0 < \omega < 1$), one can find a two-stage recursive formulation of this problem by combining the one from the old and from the young (see Appendix for more details):

$$
\begin{aligned}
\{c_{Y,0}, c_{O,1}, g_0\} &= \arg\max \{(1-\omega) q(c_{Y,0}, g_0) + \omega v(c_{Y,0}, c_{O,0}, g_0) && \text{(C.13)}\\
&\quad +\beta \widetilde{\omega} V_O^{Comm}(c_{O,1})\} && \text{(C.14)}\\
&\quad V_O^{Comm}(c_O) = \max_{c_Y, c'_O, g} \{v(c_Y, c_O, g) + \lambda\beta V_O^{Comm}(c'_O)\} \text{ for } t>0\\
&\quad \text{s.t. } g = w - c_Y - c_O
\end{aligned}
$$

where $v(.)$ is defined as above, $q(c_{Y,0}, g_0) = u(c_{Y,0}) + \theta u(g_0)$ and $\widetilde{\omega} = (1 - \omega + \omega\lambda)$. The solutions to this problem are the following (see Appendix for more details):

$$
c_{Y,t} = \begin{cases} \dfrac{\widetilde{\omega}^{\frac{1}{\sigma}}}{[\theta(1+\omega\lambda)]^{\frac{1}{\sigma}} + \widetilde{\omega}^{\frac{1}{\sigma}}}(w - c_{O,t}) & \text{in } t=0 \\ \dfrac{\lambda^{\frac{1}{\sigma}}}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}} + 1 + \lambda^{\frac{1}{\sigma}}\right]} w & \text{in } t>0 \end{cases}
$$

$$
c_{O,t+1} = \begin{cases} \dfrac{1}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}} + 1 + \lambda^{\frac{1}{\sigma}}\right]} w & \text{for } (\lambda > 0 \text{ or } \omega \neq 1) \text{ and } \beta > 0, & \text{in } t=0 \\ \dfrac{1}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}} + 1 + \lambda^{\frac{1}{\sigma}}\right]} w & \text{for } \lambda, \beta > 0 & \text{in } t>0 \end{cases}
$$

Again note that the debt level and private consumption from period $t > 0$ onwards do not depend on who decides. The reason is that the old (through altruism for their children) and the young (directly) share the same interests from $t > 0$ onwards as explained above. In contrast, private consumption in period t=0 reacts differently to inherited debt, because in this period the interests of young and old differ. Furthermore, (provided that $\beta > 0$) note that the demand for bonds is indeterminate in period $t = 0$, if there is no altruism ($\lambda = 0$) and only the old decide ($\omega = 1$) for reasons already discussed above. Logically, for subsequent periods the demand for bonds is indeterminate, if there is no altruism, regardless of the power of the old, $\omega$. One can show that for a given initial debt level, the consumption of the young in the first period is bigger the higher the power of the young (see Appendix for a proof).

The commitment solution will now serve as a benchmark of comparison to the political equilibrium, where the government cannot commit to an entire time path of the policy variables, but is reelected every period.

### C.2.4 The Political Equilibrium

Following Song et al. (2009) I assume a probabilistic voting model, where candidates or parties propose a policy platform, characterized by some policy variables (here: $g_t$, $\tau_t$ and $b_{t+1}$), but they can differ in some other dimension unrelated to this policy, e.g. "ideology". Voters differ in their valuation of this other dimension. Lindbeck and Weibull (1987) have shown that under such conditions the political choice is equivalent to maximizing a weighted objective of the indirect utilities, where the weights represent the fraction of the population with these particular preferences. Therefore, in this context, the equilibrium of the probabilistic voting model can be represented as the choice over time of $g_t$, $\tau_t$ and $b_{t+1}$ maximizing a political objective function, which is a weighted average of young and old households, given the state variable $b_t$. Similar to the commitment case the political objective function $V$ assigns a weight $\omega$ to the old and a weight $(1-\omega)$ to the young, such that if $\omega = 1$ the power is entirely in the hands of the old and if $\omega = 0$ the young have the power. Again I use a "primal approach" formulating everything in terms of consumption allocations instead of the policy variables and I take $c_{O,t} = b_t$ as a state variable (bold letters denote sequences starting from the indicated period):

$$V(\mathbf{c}_{Y,t}, \mathbf{c}_{O,t}, \mathbf{g}_t) = (1-\omega)U_Y(\mathbf{c}_{Y,t}, \mathbf{c}_{O,t+1}, \mathbf{g}_t) + \omega U_O(\mathbf{c}_{Y,t}, \mathbf{c}_{O,t}, \mathbf{g}_t) \tag{C.15}$$

where

$$U_Y(\mathbf{c}_{Y,t}, \mathbf{c}_{O,t+1}, \mathbf{g}_t) = u(c_{Y,t}) + \theta u(g_t) + \beta U_O(\mathbf{c}_{Y,t+1}, \mathbf{c}_{O,t+1}, \mathbf{g}_{t+1})$$
$$U_O(\mathbf{c}_{Y,t}, \mathbf{c}_{O,t}, \mathbf{g}_t) = u(c_{O,t}) + \theta u(g_t) + \lambda U_Y(\mathbf{c}_{Y,t}, \mathbf{c}_{O,t+1}, \mathbf{g}_t)$$

There is commitment in the sense that the promised policy will always be taken out by the candidates once in office. The lack of commitment in this model stems from the fact that the government today cannot commit future governments to any specific actions. Each political candidate will maximize his objective function with respect to the government budget constraint. The present government cannot commit future governments to any actions, but by setting the level of government debt the government today can nevertheless have an important influence on the next government in place by restricting its budget. This is important because part of the electorate cares about the actions of the future government as well (the young). Finding the political equilibrium involves finding the outcome of a dynamic game between generations of voters. Similarly to Song et al. (2009) I focus on Markov perfect equilibria of this game. In this case the only payoff-relevant state variable is consumption of the old (here denoted by $c_{O,t}$). The following definition summarizes the properties of such an equilibrium:

**DEFINITION C.5.** *A Markov Perfect Equilibrium is a 3-tuple of functions* $\langle C_O, C_Y, G\rangle$, *where* $C_O : [0, w] \to [0, w]$ *is a function showing the dynamics of debt (or equivalently consumption of the old),* $c'_O = C_O(c_O)$, $C_Y : [0, w] \to [0, w]$ *is a mapping between last period's debt and private consumption today,* $c_Y = C_Y(c_O)$, *and* $G : [0, w] \to [0, w]$ *is a mapping between last period's debt and government expenditure,* $g = G(c_O)$, *such that:*

1. $< C_O(c_O), C_Y(c_O), G(c_O) >= arg\max V(\mathbf{c}_O, \mathbf{c}_Y, \mathbf{g})$, *s. t. (C.6) and (C.8),*
   *and* $V(\mathbf{c}_O, \mathbf{c}_Y, \mathbf{g})$ *is defined as in (C.15).*

2. *The government budget constraint is satisfied:*

$$G(c_O) = w - C_Y(c_O) - c_O$$

Intuitively each government determines the current fiscal policy constraint by the government budget and expecting that governments in the future will behave according to the equilibrium policy functions, $< C_O(c_O), C_Y(c_O), G(c_O) >$ defined above. It can be shown that this problem translates to the following two stage recursive formulation similar to the one for the commitment case:

$$\begin{aligned}
&\langle C_O(c_O), C_Y(c_O), G(c_O)\rangle = \arg\max_{c'_O, c_Y, g} (1-\omega)q(c_Y, g) + \omega v(c_Y, c_O, g) \\
&+ \beta\widetilde{\omega} V_O(c'_O) \qquad\qquad \text{(C.16)} \\
&V_O(c'_O) = v(C_Y(c'_O), c'_O, G(c'_O) + \beta\lambda V_O(C_O(c'_O)) \\
&\text{s.t. } G(c_O) = w - C_Y(c_O) - c_O
\end{aligned}$$

where $v(.)$, $q(.)$ and $\widetilde{\omega}$ are defined as above.

**PROPOSITION C.4.** *Suppose we have a closed OLG economy as described above with $\beta > 0$, $\lambda > 0$ and $0 < \omega < 1$, where the private consumption of the agents is given by (C.5) and (C.4), the bond market clears and in each period the government maximizes (C.15) subject to (C.6) and (C.8). Then there exists a Markov Perfect Equilibrium characterized by the following policy functions:*

$$c_{Y,t} = C_Y(c_{O,t}) = \frac{1}{1+\xi}(w - c_{O,t}) \tag{C.17}$$

$$g_t = G(c_O) = \frac{\xi}{1+\xi}(w - c_{O,t}) \tag{C.18}$$

$$c_{O,t+1} = C_O(c_{O,t}) = \frac{(1+\xi)^{\frac{1-\sigma}{\sigma}}}{(1+\xi)^{\frac{1-\sigma}{\sigma}} + [(1+\lambda)\theta\xi^{1-\sigma} + \lambda]^{\frac{1}{\sigma}}} w \tag{C.19}$$

$$\textit{where } \xi = \left(\frac{\theta(1+\omega\lambda)}{\tilde{\omega}}\right)^{\frac{1}{\sigma}}$$

(All proofs are relegated to the Appendix).

In the next section I will discuss the results found in this section more in detail.

# C.3 Discussion of the Results

The political economy model presented above although simple and stylized yields some interesting insights regarding the influence of institutions on the determination of government debt or the reason for differences in government debt levels between different countries. In Section C.3.1, I contrast the commitment equilibrium and the political equilibrium to investigate the implications of each institutional setting for the determination of government debt. In Section C.3.2, I analyze the comparative statics of the model to see how differences in preferences can yield different levels of government debt. Furthermore, in Section C.3.3, I compare the results of this closed economy model to those of the small open economy model by Song et al. (2009).

## C.3.1 Commitment vs. Markov Equilibrium

The choice of an institutional setting such as between a democracy with periodical reelections or a commitment device (for example a constitutional debt rule) can have an important influence on the determination of public debt in an economy. In this paper, I focus on comparing those two diametrically opposed institutional settings: a determination of policies through a government that is reelected every period (Markov or political equilibrium) or through a rule that is set in the first period by the first generation of voters (commitment equilibrium). To illustrate the results I choose the following parameter values: $\beta = 0.985^{30}$, $\lambda = 0.6$, $\omega = 0.5$, $\theta = 1.00$, $w = 1$. Figure C.1 shows Markov and Commitment equilibrium policy functions with different values for the parameter $\sigma$ (from the second period onwards).

Panel (a) and panel (b) of Figure C.1 show that taxes and public goods are generally higher under commitment for any given debt level. In the commitment case the first generation that decides does not pay taxes in the future, but still profits from public goods. Thus the only consideration that prevents them to set the maximum tax rate is the altruism for their children. In

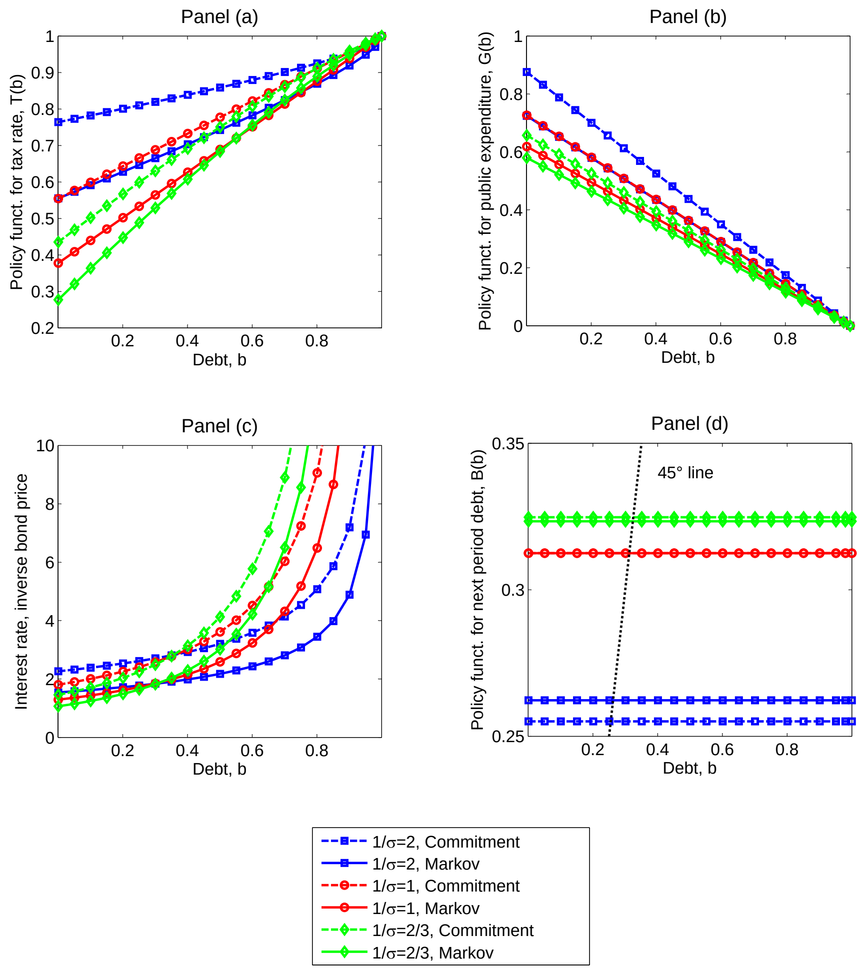


**Figure C.1:** Comparison of Markov and Commitment Equilibrium

contrast in the political equilibrium each generation decides themselves how much taxes they want to set for themselves. Thus they have a direct interest in setting lower taxes and more moderate public goods rather than indirect through altruism.

A high debt burden can be financed in both cases either by taxation or by reducing public goods provision. Starting from an already high level of taxes and public goods in the commitment case the reaction will therefore rather go in the direction of reducing public goods. Therefore the policy function of taxes is flatter and the policy function for public goods is steeper in the commitment case compared to the political equilibrium.

To understand the intuition behind those results consider therefore next the determination of next-period's debt level in Panel (d) of Figure C.1. The steady state level of government debt is given at the intersection of the policy function with the 45-degree-line. A noticeable feature of the policy functions for debt is that they are flat, i.e. independent of the previous level of government debt. As already brought forward in the discussion of the commitment solution this is a generic feature of this model of closed economy. There is a intertemporal dichotomy in the model where government debt is only relevant for the next period's allocation of resources between public and private consumption and the choice of the tax rate (or equivalently consumption of the young) is only relevant for this period's allocation of resources between public and private consumption. Thus there is no intertemporal link and the economy jumps to the equilibrium debt level in one period.

The equilibrium debt level is thus determined by intratemporal considerations about how much public and private consumption there should be for the old generation next period (this period's young). Those intratemporal considerations apply under commitment and in the political equilibrium. However, in the political equilibrium there is additionally a 'strategic effect' of public debt on the next generation of young voters. Young voters tomorrow react to a higher debt level left by today's generation by increasing the taxes they set themselves and reducing their private consumption, as can be seen from Panel (a) in Figure C.1. The reason is that a higher level of government debt restrains next period's budget and lowers the amount of public goods that can be financed. To avoid too large a loss of public goods the young are ready to reduce their private consumption (or equivalently tax themselves more). The young today know about this reaction and can make use of it by deviating from the level of government debt that solves optimally the tradeoff between public and private consumption in old age. They can enforce a level of consumption of the old that is somewhat higher to rip the young tomorrow off their resources. Under commitment, of course, they don't need to deviate from their optimal debt level because they can directly control consumption of the young in the next period. It is thus enlightening to compare the first order condition for debt under commitment and in the political equilibrium, for the simple case when there is no altruism $\lambda = 0$:

$$\begin{aligned} \text{Commitment:} \quad & u'(c_O') - \theta u'(w - c_O' - c_Y') = 0 \\ \text{Political equilibrium:} \quad & u'(c_O') - \theta u'(w - c_O' - C_Y(c_O'))\,[1 + C_Y'(c_O')] = 0 \end{aligned}$$

The conditions look very similar except for the fact that in the political equilibrium the reaction of the policy function of private consumption, $C_Y(.)$, on the debt level, $c_O' = b'$, plays a role. This formally shows the 'strategic effect' discussed above.

In the light of those considerations one can understand the role of the endogenous interest rate or bond market in the model. Panel (c) of Figure C.1 shows the equilibrium interest rate (or inverse price of bond) for any given debt level. Remarkably high given debt levels lead to very high interest rates. This is quite intuitive considering the results above. A high debt level leads to an increase in taxation and a decrease in public goods provision. The disposable income of today's young generation is thus restrained. The government (the planner or the voters) wants to set public debt in a certain way to solve tomorrow's tradeoff between public and private consumption. But the young household's of today with low disposable income have naturally a lower demand for bonds. Thus the government through the national bond market is forced to offer high interest rates to be able to sell enough bonds.

Interestingly, the determination of next-period's debt level depends on the elasticity of substitution. A 'knife-edge' case exists for logarithmic utility:

**PROPOSITION C.5.** *Under logarithmic utility, i.e. if the elasticity of substitution goes to* 1*:* $\frac{1}{\sigma} = 1$*, the level of next-period's government debt is equal in the political equilibrium and in the commitment equilibrium. For* $\frac{1}{\sigma} > 1$ *the level of next-period's government debt is higher in the political equilibrium and for* $\frac{1}{\sigma} < 1$ *the level of next-period's government debt is lower in the political equilibrium.*

Note that $\frac{1}{\sigma}$ fulfills a dual role in the model, being at the same time the elasticity of substitution between public and private goods and the elasticity of intertemporal substitution. Given the intertemporal dichotomy of the model the intertemporal elasticity of substitution doesn't play any role for the determination of the level of government debt. For the strength of the strategic effect, the parameter $\sigma$ is thus especially important in its function as a parameter of substitutability between public and private goods.

It is not completely clear in the macroeconomic literature what the elasticity of substitution between public and private goods should be. There are examples of public goods which would rather be seen as complements to private goods (like infrastructure or the legal system) and others which would rather be seen as substitutes (such as health or education).

Nevertheless there is a large empirical literature estimating the elasticity of substitution between public and private goods (see also, for example, Bandyopadhyay and Esteban, 2007, p.22, for a discussion of this literature). Early studies analyzing this question defined substitutability by the value of the cross derivative between public and private consumption. Using this measure of substitutability Kormendi (1983) and Aschauer (1985) for the United States and Ahmed (1986) for the United Kingdom find evidence that public and private goods are substitutes. In contrast, Karras (1994) for 30 countries and Evans and Karras (1996) for 54 countries provide evidence from cross-country panel regressions that public and private goods are complements. For Japan Hamori and Asako (1999) find substitutability whereas Okubo (2003) finds complementarity or no relationship. Comparing all those results, they seem disappointingly contradictory. Furthermore it seems that they are not robust to the exact specification of the utility function, as prominently shown by Ni (1995).[2] This casts doubt on the generality of any empirical estimates of the elasticity of substitution between public and private goods using the cross-derivative. Furthermore this way of measuring substitutability is not the same as in this paper, which considers not the cross-derivative, but the elasticity of substitution.

Corresponding most closely to the concept of substitutability used in this paper Amano and Wirjanto (1998) estimate an elasticity of substitution between public and private goods of around 1.56 in the US (slightly substitutes). Even if this concept of elasticity seems a little more general than the cross-derivative, Bouakez and Rebei (2007) find that even those estimates are not robust to the time-separability of utility.[3]

How does the elasticity of substitution determine the strength of the strategic effect? To use government debt strategically the young agents today have to deviate from the level of government debt that optimally solves their tradeoff between public and private consumption tomorrow. The higher the elasticity of substitution the less costly it is to deviate from this optimal tradeoff level. Note also that under commitment as the future tax rate is generally higher (it can be set by the first generation), the optimal tradeoff level of government debt is higher than in the political equilibrium. It turns out that, if

[2]More precisely, he finds that estimates change depending on the time-separability of utility, the measure of the real interest rate and how public and private consumption enter the utility function (additively separable or not).Under time-separability of the utility function estimates indicate substitutability. Under time-nonseparability and using the net-of-tax interest rate there is still substitutability whereas using the pre-tax real interest rate estimates indicate complementarity. Under a CES-specification (non additively separable public and private consumption) there is however complementarity with the net-of-tax real interest rate and substitutability with the pre-tax real interest rate.

[3]Their estimate of the elasticity of substitution between public and private goods assuming a utility function displaying habit-formation is equal to 0.332. Effectively habit-formation is the form of non-timeseparability considered also by Ni (1995).

$\frac{1}{\sigma} = 1$ the debt level under commitment (optimal tradeoff level) is exactly the same as the debt level in the political equilibrium (optimal tradeoff level plus strategic effect), as shown in Proposition 2. It is thus not necessarily the case that a commitment device leads to lower debt levels than democratic voting, even if strategic effects play a role. If public and private goods are substitutes, government debt is higher in the political equilibrium than under commitment. In this case a commitment device would make sense to avoid a too high debt burden. If public and private goods are substitutes, government debt is lower in the political equilibrium than under commitment. In this situation a commitment device which is put into place by the first generation of voters, leads to a higher debt level.

A commitment or a voting institution are not the only possible differences between countries that can have an important influence on the politico-economic determination of government debt in this model. The next section analyzes the influence of the underlying preference parameters in a comparative statics analysis.

### C.3.2 Comparative Statics

Large differences in public debt between countries, even inside the group of democratic countries, suggest that the determination of public debt depends, on top of the form of government, on cultural and institutional characteristics that differ between countries. Those cultural and institutional characteristics can be captured in the context of this model by the underlying preference parameters.

The parameter summarizing the preference for public goods relative to private goods, $\theta$, shows the concern of the voters for public goods provision or their view about how much government activity there should be in the country. The parameter for altruism, $\lambda$, shows the concern of voters for future generations and the parameter for the power of the old relative to the young,

$\omega$, shows the political influence of the old generation relative to the young generation. All those parameters can influence of course the determination of the level of public debt in a country.

Depending on the strength of the strategic effect, the preference parameters can have differential effects on the determination of government debt. As shown in Proposition 2 above, the elasticity of substitution between public and private goods $\frac{1}{\sigma}$ is decisive for the strength of the strategic effect. Interesting for the analysis here is the empirical question, if this elasticity of substitution is equal for different countries. Kwan (2006) provides evidence that this is not the case even within the group of East-Asian countries. Bandyopadhyay and Esteban (2007) argue that the substitutability between public and private goods "depends on the nature of individual preferences and on the degree of monopoly that the government keeps for itself for some subset of commodities ..." (Bandyopadhyay and Esteban, 2007, p.22). Using OECD data they estimate a proxy for the elasticity of substitution in different sectors (primary, secondary, tertiary education and health) and show that there are important differences even within the group of OECD countries.[4] Thus I will analyze both cases here, complementarity and substitutability.

### Complementary Public and Private Goods

For the case of complementarity ($\frac{1}{\sigma} < 1$) the following comparative static effects on the level of next-period's government debt in the political equilibrium arise (more details in the Appendix):

1. The higher the power of the old, $\omega$, (or equivalently the lower the power of the young, $1 - \omega$), the higher is next period's debt.
2. The higher the degree of altruism, $\lambda$, the lower next-period's debt.
3. The effect of the preference for public goods, $\theta$, is ambiguous.

[4]For example, in the tertiary eduction sector and for health the measure indicates complementarity in some countries but substitutability in others.

Those comparative static effects clearly relate to the general intuition about a 'strategic effect' discussed in section C.3.1 above.

First, the higher the voting power of the old generation the nearer the debt level approaches the one under commitment (which is higher). The reason is that the strategic effect is then less of an issue. The young generation today do not need to set a high debt level as they can directly vote for higher taxes tomorrow. Of course, in the extreme case when the old have all the power, $\omega = 1$, the debt level is the same as under commitment, because they do not need the strategic effect and can set tax rates themselves. Effectively, as the power of the old increases the political equilibrium approaches the commitment solution.

Second, a higher altruism parameter $\lambda$ leads unambiguously to a lower next-period's debt level. Intuitively, under commitment public goods are more attractive for altruistic parents, as they also profit to their children. Thus even the desired level of public debt is lower with higher altruism, as parents want to leave more resources for public goods provision and consumption of their children. In the political equilibrium, in addition to the fact that the desired level of debt in the view of the altruistic parents is already lower, a higher altruism in addition limits the use of the strategic effect to achieve a higher public goods provision. The more altruistic parents do not want to rip off their children to the same extent.

Third, a higher preference for public goods, $\theta$, has an ambiguous effect on the level of public debt next period. Intuitively, under commitment the young today would unambigously prefer a lower level of private vs. public consumption tomorrow as their preference for public consumption increases. However, in the political equilibrium a higher preference for public goods has also the effect of dampening the strategic effect on the next generation. Public goods will already be higher, because also young voters tomorrow

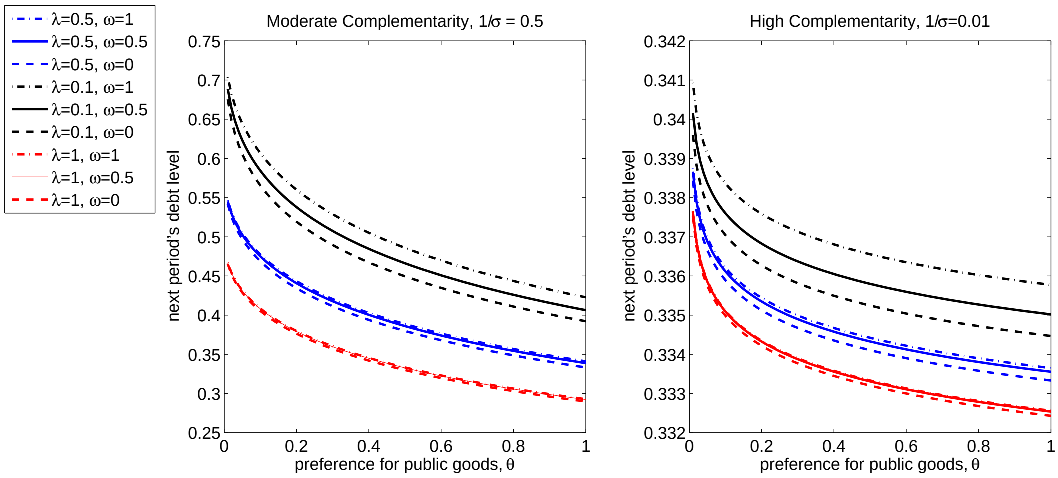


**Figure C.2:** The Influence of the Preference for Public Goods on the Determination of Government Debt (Case of Complementarity)

prefer more public goods. The conjecture is however that as long as there is altruism, which produces a higher value of the public good as it is shared, a higher preference for public goods should nevertheless decrease further the level of public debt. Figure C.2 confirms this intuition. Note however that for high complementarity the curves are very flat.

### Substitutable Public and Private Goods

For the case of substitutability, $\frac{1}{\sigma} > 1$, one can distinguish the following comparative static effects on next-period's debt level:

1. The higher the power of the old, $\omega$, (or equivalently the lower the power of the young, $1 - \omega$), the lower is next period's debt.

2. The higher the preference for public goods, $\theta$, the lower is next-period's debt.

3. The effect of the degree of altruism, $\lambda$, is ambiguous.

First, similar to before the higher the power of the old the more the solution will resemble the commitment solution. Intuitively, the young generation today does not need to make use of the strategic effect, because they can directly use their voting power tomorrow. Thus the debt level will decrease, the higher the power of the old.

Second, the effect of a higher preference of public goods is clear now. As public goods become more attractive not only does the desired level of debt decrease, but in addition the strategic effect is dampened. Thus the debt level with decrease unambiguously with a higher preference for public goods in this case.

Third, the effect of altruism is ambiguous in the case of substitutability. But given that the effect on the desired level of debt is negative, the conjecture is that the effect will also be negative in the political equilibrium for any reasonable parameter values. Figure C.3 confirms this intuition.

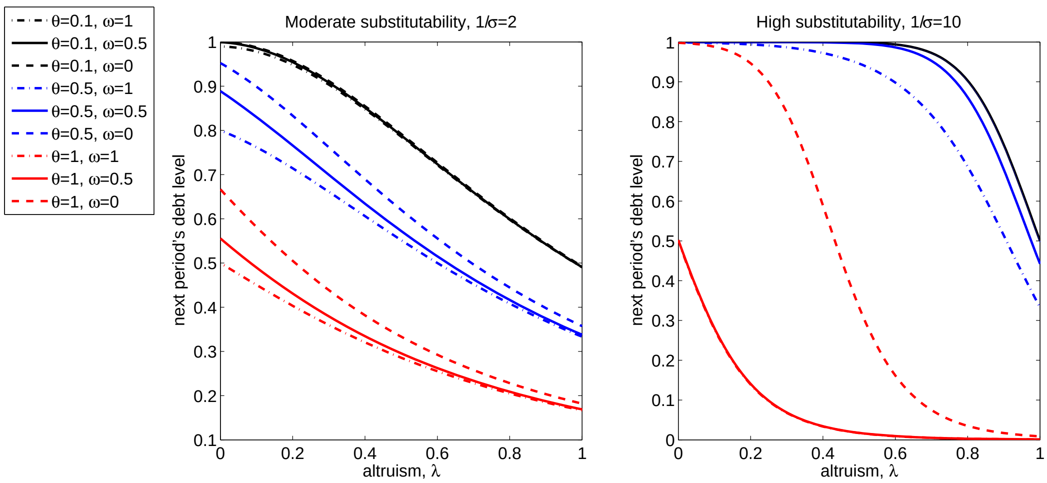


**Figure C.3:** The Influence of the Degree of Altruism on the Determination of Government Debt (Case of Substitutability)

Summarizing, the degree of altruism and the preference for public goods seem to dampen a potential tendency of high indebtedness according to this model. One would thus expect to see lower public debt in countries where public goods are viewed as being more important or where the concern for future generations is higher. The effect of political power of old vs. young agents in the economy depends on the degree of substitutability between public and private goods. When public and private goods are complements a higher power of the old generation leads to higher public debt. When public and private goods are substitutes, the higher the voting power of old voters the lower the level of public debt.

Besides those underlying preference parameters, another characteristic of a country is its degree of openness. The next section compares the open and the closed economy models to understand the role of openness for the determination of public debt.

### C.3.3 A Comparison between the Open and Closed Economy Models

Although today's financial markets are supposedly open to the participation of any country and not confined to national markets, it is a well known puzzle that in reality still most of the activity stays inside the national markets of each country. In fact, it is sort of a stylized fact in international macroeconomics that despite the openness of financial markets the correlation between savings and investment inside a country is still high for a lot of countries (as first noted by Feldstein and Horioka, 1980). Thus in today's economies a change in the supply of bonds can still have an effect on the interest rate on those bonds, although it is probably not as extreme as if the economy was completely closed. It thus makes sense to analyze a model with an endogenous interest rate as well as a small open economy set-up with an exogenous interest rate and contrast the results, as the reality probably lies somewhere in between. In this section, I will thus compare the results of the model presented above to the small open economy model of Song et al. (2009).

The definitions of government debt in the two models are slightly differ-

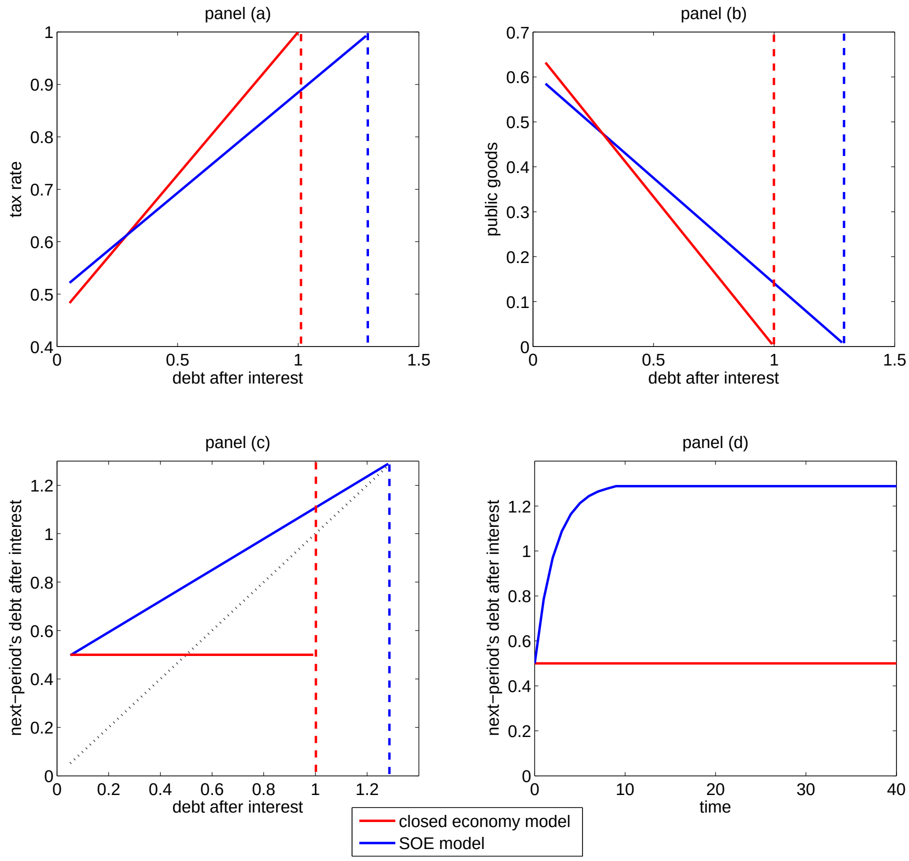


**Figure C.4:** Comparison of Policy Functions for the Closed and Open Economy

ent. Whereas Song et al. (2009) define debt as debt before interest, I have chosen to define it as debt after interest, which in the closed economy context is practical as it is equal to consumption of the old. To undertake the comparison I therefore need a common denominator, which I choose to be my definition, debt after interest. The illustrative parameters are set like in Example 1 of Song et al. (2009): $\beta = 0.985^{30}$, $\lambda = 0$, $R = 1.05^{30}$ (for the gross interest rate or equivalently for the price of bond$p \approx 0.23$), $\omega = 0.5$, $\theta = 1.00$, $w = 1$ and $\sigma = 1$ (log-utility). Figure C.4 shows how the policy functions in terms of debt compare in the two models:

Panel (a) and (b) show that the policy functions for taxes and public goods are very similar. In both cases a higher debt burden is financed partly by a higher tax rate and by a lower level of public goods. However, as shown in panel (c), the policy function of public debt is very different. Whereas in the open economy public debt can be used to shift resources from tomorrow to today, by borrowing on the international financial market, in the closed economy no resources can be shifted but the resource constraint has to hold, so that public debt is only relevant for the allocation of different kinds of consumption tomorrow. Thus public debt is independent of the state of the economy today as already argued above. Therefore whereas next-period's debt is used to finance higher expenditures in the face of a high given debt burden today in the open economy model, it is independent of the state today in the closed economy model. Of course this translates to very different dynamics of public debt, shown in panel (d). Whereas public debt is constant in the closed economy it is increasing over time until it reaches the maximum in the open economy.

Note also that the upper bound for debt is different in the two economies. The reason is that the open economy is limited by the present discounted value of the maximum tax revenue that can be collected:

$$\bar{b}_{open} = \frac{R\bar{\tau}w}{(R-1)} \approx 1.3$$

where $R = 1.05^{30}$, $w = 1$ as specified above. $\bar{\tau}$, the maximal tax rate is equal to 100% in the case of inelastic labor supply.

In contrast, in the closed economy debt after interest (or equivalently consumption of the old) is limited by the maximal amount of consumption of the old that is possible, i.e. the endowment of resources, $w$:

$$\bar{b}_{closed} = \bar{c_O} = w = 1$$

Note that this debt limit is tighter than the one of the open economy. Intuitively there is more room for issuing debt in an open economy because the government can borrow from an international capital market with deep pockets, whereas in the closed economy a high borrowing necessarily involves distorting the consumption allocations of the agents in the economy.

Concluding one can say that the analysis of this section clearly predicts a lower long run level of government debt in closed vs. open (democratic) economies in the case of no distortive taxation. This result can be easily seen to hold for the wide range of sensible parameter values.[5] This finding suggests a potential explanation for why debt levels increased so much in a lot of today's developed economies during the last decades. It might be related to financial globalization which makes borrowing abroad much easier than if the bonds have to be sold inside the country. However one must add the caveat that, as shown by Song et al. (2009), for the case of distortive taxation also the open economy model reaches an interior steady state which is much lower than the maximum. Therefore the above argument would only make sense as an explanation for countries where tax distortions are absent or very small. Otherwise the simple fact of financial openness should not push to high long run debt.

To illuminate the role of tax distortions for the determination of public debt further I will thus analyze in the next section an extension of the model where labor supply is elastic and thus tax distortions can arise.

[5] In fact, even for the extreme case of $\theta = 0$ where only private goods are consumed and thus in the closed economy the maximum debt level of $w$ is reached, it would in addition take a negative interest rate (or a gross interest rate $R < 1$) to reach a smaller debt level in the open economy in the long run.

## C.4 An Extension: A Model with Elastic Labor Supply

In the baseline model it was assumed that labor supply is inelastic. This assumption allows me to solve the model analytically using a primal approach as shown above. However this is of course a crude simplification of reality. In the real world we observe that economic agents react to higher taxes on the labor supply to some extent by working less. The distortions arising from this endogenous reaction of labor supply could change the way a policy is chosen. It is therefore important to check if the results of the model still hold when labor supply is somewhat elastic.

### C.4.1 Household Production

The way I introduce an elastic labor supply is inspired by Song et al. (2009). For this approach it is assumed that the household can either work in the market production sector and subsequently buy goods in the goods market from the wage income earned, $w$, or produce goods directly via home production using the technology $y_H = F(h) = X(1 - h^{1+\xi})/(1+\xi)$, where $h$ is hours worked in the market production, $\xi > 0$ is the inverse of the Frisch elasticity, $X$ is a constant defining the value of leisure. There is a linear tax $\tau \in [0, 1]$ on wages. It is thus implicitly assumed that the government cannot tax home production. Therefore a high tax distorts the time agents work in the market production sector. The agents choose their allocation of time so as to maximize total after-tax income:

$$A(\tau) \equiv \max_{h \in [0,1]} \{(1-\tau)wh + F(h)\}$$

This maximization problem yields labor supply as a function of the tax rate:

$$h(\tau_t) = \left( \frac{(1-\tau_t)w}{X} \right)^{\frac{1}{\xi}}$$

The solutions to the private optimization problem are exactly the same as before, but using the new definition of after tax income $A(\tau_t)$. For simplicity I assume log-utility ($\sigma = 1$). In this case the solutions of the private optimization problem are simply given by:

$$\begin{aligned} c_{Y,t} &= \frac{1}{1+\beta} A(\tau_t) && \text{(C.20)} \\ c_{O,t+1} &= \frac{1}{p_t} \frac{\beta}{1+\beta} A(\tau_t) = b_{t+1} && \text{(C.21)} \end{aligned}$$

## C.4.2 The Government Constraints

The budget constraint of the government now involves the reaction of hours to the tax rate:

$$p_t b^s_{t+1} = g_t + b_t - \tau_t w h(\tau_t) \tag{C.22}$$

Again we can transform the government budget constraint into a resource constraint. Output (or resources) in the economy however now involves both home and market production:

$$\begin{aligned} Y(\tau_t) &= wh(\tau_t) + X(1 - h(\tau_t)^{1+\xi})/(1+\xi) \\ &= A(\tau) + \tau_t w h(\tau_t) \end{aligned}$$

Output is a function of the tax rate, because a higher tax rate influences output through the hours allocation between the market and home production sectors. The second part of the equation shows that output can also be written in terms of after tax income and tax revenues (by simply adding and subtracting tax revenues). Assuming bond market clearing $b^s_{t+1} = b^d_{t+1}$ and using the first order conditions in equation (C.20) and (C.21) one can eliminate the price of bond in equation (C.22) and obtain a formulation similar to a resource constraint:

$$g_t = Y(\tau_t) - \frac{1}{1+\beta} A(\tau_t) - b_t$$

In this extended model there is a new debt limit which is stricter than simply the maximum amount of resources. The reason is that home production cannot be used to fulfill debt liabilities as it is not taxable. Therefore from the requirement of positive public goods $g > 0$ follows that a new debt limit is now given by the maximum amount of tax revenues that can be achieved:

$$\bar{b}_{new} = \bar{\tau} w h(\bar{\tau}) \tag{C.23}$$

where $\bar{\tau}$ in this model denotes the top of the Laffer curve.

### C.4.3 The Commitment Problem

In this economy with tax distortions the "primal approach" is not practicable because one would have to solve for the tax rate in terms of allocations, but it is only implicitely given. The commitment problem thus has to be cast in terms of policy variables, like a Ramsey planning problem, by substituting in the private solutions in equation (C.14) (now formally denoting the state variable by $b_t$ to signal that the Ramsey approach is used):

$$\begin{aligned}
&\{\tau_0, b_1, g_0\} = \arg\max \{(1-\omega)\hat{q}(\tau_0, g_0) + \omega\hat{v}(\tau_0, b_0, g_0) \\
&+\beta\widetilde{\omega} V_O^{Comm}(b_1)\} && \text{(C.24)} \\
&V_O^{Comm}(b) = \max_{\tau, b', g} \left\{\hat{v}(\tau, b, g) + \lambda\beta V_O^{Comm}(b')\right\} \text{ for } t > 0 \\
&\text{s.t. } g = Y(\tau) - \frac{1}{1+\beta} A(\tau) - b && \text{(C.25)}
\end{aligned}$$

where $\hat{v}(\tau, b, g) = u(b) + (1+\lambda)\theta u(g) + \lambda u(A(\tau))$, $\hat{q}(\tau, g) = u(A(\tau)) + \theta u(g)$ and $\widetilde{\omega} = (1 - \omega + \omega\lambda)$. Note that the debt limit does not have to be imposed for the maximization problem, because public goods consumption will be automatically be chosen to be above zero. However the state variable has to be defined inside the bounds $b \in [0, \bar{b}_{new}]$ otherwise feasibility is not given.

### C.4.4 The Political Equilibrium

Also for the political equilibrium the "primal approach" is not possible anymore. Therefore we again have to substitute in the private solutions and use a similar two stage recursive approach as in equation ($C$.16) searching for the policy functions $B(b)$, $T(b)$ and $G(b)$ now:

$$
\begin{aligned}
&< B(b), T(b), G(b) >= \arg\max_{b',\tau,g}(1-\omega)\hat{q}(\tau, g) + \omega\hat{v}(\tau, b, g) + \beta\widetilde{\omega}V_O(b') \\
&V_O(b') = \hat{v}(T(b'), b', G(b') + \beta\lambda V_O(B(b')) \\
&\text{s.t. } G(b) = Y(T(b)) - \frac{1}{1+\beta}A(T(b)) - b
\end{aligned}
$$

where $\hat{v}(.)$, $\hat{q}(.)$ and $\widetilde{\omega}$ defined as above for the commitment problem.

### C.4.5 Benchmark Calibration

The model has to be solved numerically. I use a standard grid search to find the maximum of the objective functions shown above.[6] The calibration strategy is inspired by Song et al. (2009). One period is set to thirty years and I assume log-utility. However as I have a closed economy model and debt is equal to savings of the old, debt will generally be quite high in the political equilibrium as it constitutes the only savings device. Therefore instead of targeting the US debt-to-GDP ratio I target an annual savings-to-output ratio of 3.3. Then the strategy is to set $\lambda$, $\theta$ and $X$ to simultaneously target the savings to output ratio (=government debt here), an average steady state labor income tax rate of 28% and a market to total ratio of consumption of around 30%. With one period being thirty years, a labor share of output of 0.67 and a target for the interest rate of 2.5% the after interest steady state level of savings to labor earnings is equal to $c_O/wH = (1.025^{30}) \times 3.3 \times 0.67/30 = 0.1545$.

I assume equal political weights of young and old ($\omega = 0.5$), normalize the

[6] This method has the advantage to being quite robust and easy to implement but the precision is not that high compared to projection methods with collocation.

| Target observation | | Parameter | |
|---|---|---|---|
| Savings to labor earnings ratio (=debt) | 3.3 | $\lambda$ | 0.95 |
| Average tax on labor income | 28% | $\theta$ | 0.02 |
| Market to total consumption | 30% | $X$ | 3 |
| Political weight old vs. young | equal | $\omega$ | 0.5 |
| Tax rate at the top of the Laffer curve | 60% | $\xi$ | 1.5 |
| Equilibrium interest rate | 2.5% | $\beta$ | $0.9088^{30}$ |

**Table C.1:** Parameter values and targets for the benchmark calibration

wage to unity ($w = 1$) and the Frisch elasticity such that the top of the Laffer curve is $\bar{\tau} = \frac{\xi}{1+\xi} = 0.6$. $\beta$ is set to target an equilibrium interest rate of 2.5%. Table C.1 shows the parameters and target observations used to determine them. The parameter values presented here are somewhat different than the ones for the small open economy model used by Song et al. (2009). The discount factor, $\beta$, is smaller to achieve a high enough interest rate, which is endogenous in this model in contrast to the small open economy. $X$ is somewhat higher (3 instead of 2.7) because inducing a higher value of leisure (and thus more tax distortions) helps to achieve a lower debt level and lower tax rate. $\theta$ is even lower than in the small open economy calibration and $\lambda$ even higher. Again the reason is that it is hard to achieve at the same time a low tax rate and a low enough level of government debt in the model.[7] Not surprisingly with such a low $\theta$ the implied ratio of steady-state government expenditure to private market consumption is much too low as a consequence (2% instead of 20%).

Figure C.5 shows the equilibrium policy functions for the benchmark calibration with tax distortions ($X = 3$), comparing it to an alternative economy with exactly the same parameter values except for no tax distortions ($X = 0$).

[7]If I had chosen $X = 2.7$ as in the calibration of Song et al. (2009) even setting the maximal altruism parameter $\lambda = 1$ (which should make government debt rather small) government debt would still be too high. The downside of $X = 3$ is that the ratio of market to total consumption is only 26%, but this constitutes a compromise for matching better the ratio of savings or government bonds to labor earnings.

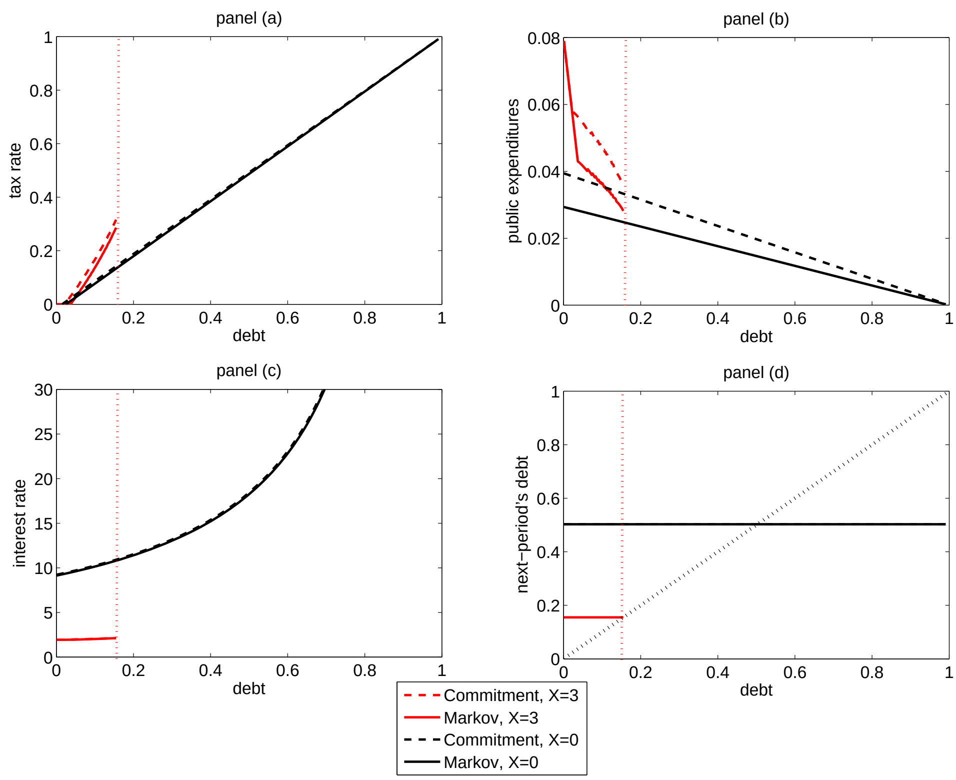


**Figure C.5:** Comparison of Commitment and Markov Equilibrium for (Benchmark Calibration)

First note, that the maximum debt limit is much lower in the case with high tax distortiveness (the red lines thus end earlier than the black lines). Second, In both cases, with and without tax distortions, the level of government debt is the same under commitment and in the political equilibrium. Note that the equilibrium level of government debt is much higher for $X = 0$ as for $X = 3$. To understand the intuition consider that as the value of home production, $X$, increases the value of taxable resources in the economy decreases and this part of resources is the only one that the old agents can access, as they don't produce themselves and depend entirely on the government (either through bond holdings or through public goods). Therefore even if the relative tradeoff between public and private goods itself remains unchanged, the old agents allocate less resources to those consumption activities the more taxation is distortive. Consequently the level of government debt falls with the level of taxable resources. This is also the reason why the maximum debt limit is much lower in the case with high tax distortiveness (dotted red line in the figure) than in the case without tax distortions (where it is equal to the endowment, $w = 1$).

The policy functions for the tax rate, panel (a), and for the public expenditures, panel (b) of Figure C.5, are qualitatively very similar in both cases. Similar to the case without tax distortions taxes are generally higher under commitment than in the political equilibrium. For very low initial debt levels there is a corner solution because even though the number of bonds are limited by the savings demand, the bond sales provide so much funds that the tax rate can be set to zero.

Summarizing one can say that the qualitative conclusions of the model regarding the policy functions are robust to the inclusion of tax distortions. The next section compares this extended model to the open economy version calibrated similarly and analyzed numerically by Song et al. (2009).

### C.4.6 Closed and Open Economies with Tax Distortions

Suppose the small open economy model of Song et al. (2009) was the benchmark calibration matching features of reality. How would the determination of debt look like in a closed economy with exactly the same underlying parameters? To answer this question, consider an alternative parametrization to the benchmark with the same parameter values as in the small open economy model of Song et al. (2009) except the discount factor which is set to target a reasonable interest rate. Table C.2 shows this alternative parametrization.

| Parameter | |
|---|---|
| $\lambda$ | 0.674 |
| $\theta$ | 0.092 |
| $X$ | 2.7 |
| $\omega$ | 0.5 |
| $\xi$ | 1.5 |
| $\beta$ | $0.9158^{30}$ |

**Table C.2:** Parameter values of alternative parametrization

Figure C.6 shows the equilibrium policy functions for the political equilibrium and the commitment case. Note that the level of government debt is much higher in the closed than in the open economy (0.052[8] in the open economy vs. 0.167 in the closed economy). In the open economy government debt is reduced by the young voters as they fear a cut in public goods provision in their old age. As the revenues from the bonds accrue mostly to people outside the country they don't have to fear necessarily lower private consumption in old age. If the young agents wanted to save more in the open economy they could just buy bonds in the international capital market. However in the closed economy the level of government debt also has this role of ensuring old age private goods consumption. Therefore the level of government debt although reduced by the distortiveness of taxation and ensuing reduction of taxable resources relative to the case of non-distortiveness, is still higher than in the open economy. The old simply require at least some private savings in their old age even if it is at the cost of some efficiency losses.

## C.5 Conclusions

In this paper I analyzed a dynamic politico-economic overlapping generations model with young and old voters and an endogenous interest rate. The role of government debt as a financing instrument is absent. Instead the focus was

[8] Song et al. (2009) report a steady state level of government debt before interest of 0.026. Given an interest rate of $1.025^{30}$ this translates approximately to a debt level after interest of 0.052.

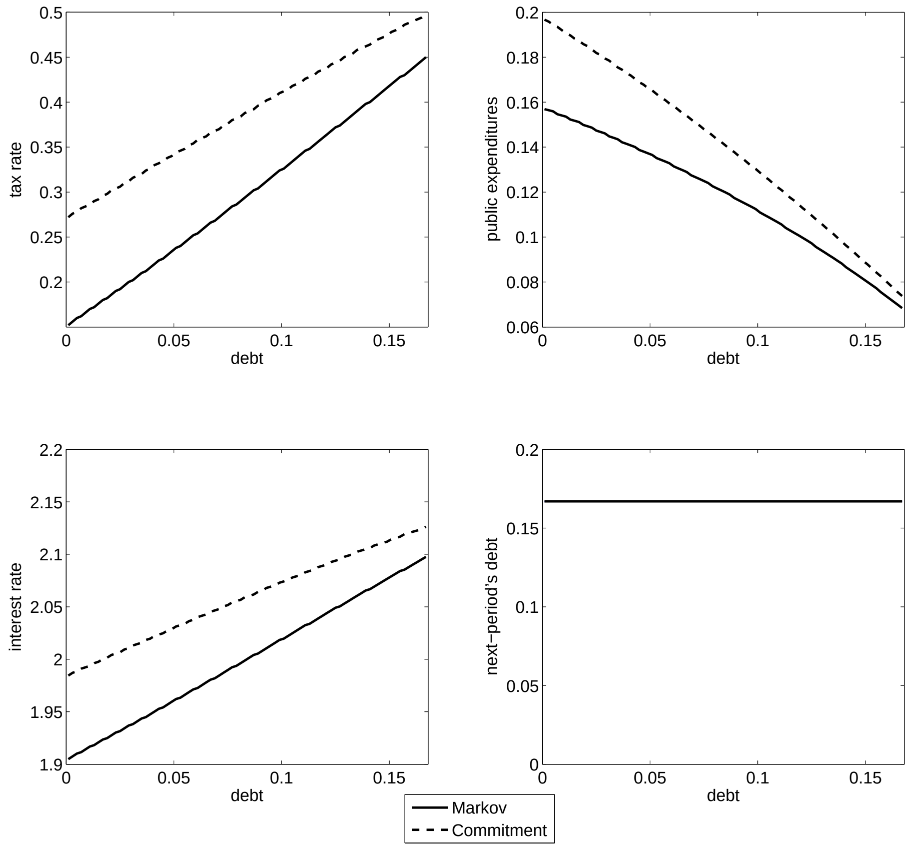


**Figure C.6:** Comparison of Commitment and Markov Equilibrium (Alternative Parametrization)

on the role of government debt as a savings instruments for households and on the strategic use of government debt to influence the decisions of future governments. Under commitment the level of government debt is used by today's young generation to obtain an optimal mix between private and public consumption tomorrow in their old age. In the political equilibrium there is additionally a strategic effect. Setting a higher debt level than the one that would satisfy their optimal tradeoff between public and private goods in old age young agents today can induce a higher taxation next period. In this way they can rip off resources from young agents tomorrow. The strength of this effect depends on the elasticity of substitution between public and private goods.

Coming back to the question asked in the beginning as to how it can happen that relatively "rich" countries exhibit a high government debt, this paper yields some first hints on the underlying causes. On the one hand government debt partly constitutes the savings of the young generation for their old age, on the other hand in a political conflict between the generations government debt can also be used as a strategic instrument to influence the next generation leading to potentially higher government debt (in the case of high substitutability between public and private goods). Another research question concerned the influence of institutional mechanisms on government debt. Here one can conclude from the results in this paper that according to this closed economy OLG framework a commitment device would lead to higher (lower) government debt than a voting mechanism in the case of complementarity (substitutability) of public and private goods. A novel aspect is the role of the elasticity of substitution to evaluate this question. To illuminate effects of different institutional mechanisms further this paper therefore suggests that it is important to consider this substitutability between public and private goods. This finding could also guide future empirical research.

Concerning the characteristics of a country that lead to high indebtedness in theory this paper has identified besides the elastiticity of substitution and the institutional mechanism of government several other factors that determine underlying preferences in a country and have an influence on the level of government debt: preference for public goods, altruism and the political power of the old. A higher preference for public goods and a higher altruism, lead to a lower level of government debt. The influence of voting power depends on the strength of the strategic effect, a higher voting power leading to a debt level nearer to the commitment level.

Finally the question of the role of financial openness of a country was analyzed by comparing the results to the ones of the small open economy set-up by Song et al. (2009). Without tax distortions, government debt is generally lower in the political equilibrium of closed economies. With high enough tax distortions, government debt can be lower in the open economies.

Of course this paper is just a first step in the direction of investigating the role of politico-economic motives for the determination of government debt with overlapping generations. Future research has to show the sensitivity or robustness of those results to possible model extensions such as the inclusion of capital accumulation, technology growth, trade, or more than two generations of voters. Furthermore, a quantitatively more meaningful set-up (hybrid between open and closed economy, more generations) could be used to quantitatively assess the importance of different factors to explain international differences in public debt levels.

# C.6 Appendix to the Paper

## C.6.1 The Commitment Solution

### More Detailed Statement of the Commitment Problem

The commitment solution is defined as a feasible time path for all variables that would be chosen if the first generation could commit all future generations to follow its will. To find an analytical solution I use a "primal" approach where the policymaker maximizes the utility of the current generation of voters with respect to consumption allocations directly taking the private first order conditions as constraints to the optimization problem.

The optimization problem in the commitment case in period 0 thus looks as follows:

$$\max_{\{c_{O,t}\}_{t=1}^{\infty},\{c_{Y,t}\}_{t=0}^{\infty}} (1-\omega)U_{Y,0} + \omega U_{O,0} \tag{C.26}$$

where $U_{Y,0}$ defined by (C.1) and $U_{O,0}$ defined by (C.2), where $g_t$ for all $t$ was substituted by using the government bugdet constraint (C.6) and $b_{t-1} = c_{O,t}$:

$$g_t = w - c_{O,t} - c_{Y,t}$$

Note that the value of consumption of the old (or the inherited debt level) in period 0 must be given as an initial value. Later on one can derive the implied government expenditures by using the government budget constraint shown above, the level of government debt by using the fact that $c_{O,t+1} = b_t$ and the tax rate can be derived by using the first order conditions and the budget constraints of the private optimization. More precisely, first find the price of bond by using the Euler equation:

$$p_t = \beta \frac{u'(c_{O,t+1})}{u'(c_{Y,t})}$$

Then use the budget constraint to find the tax rate:

$$\begin{aligned} c_{Y,t} + p_t c_{O,t+1} &= w(1-\tau_t) \\ \tau_t w &= w - c_{Y,t} - \beta \frac{u'(c_{O,t+1})}{u'(c_{Y,t})} c_{O,t+1} \end{aligned} \tag{C.27}$$

**Only the Old Decide: Recursive Formulation of the Maximization Problem**

In the case where only the old decide ($\omega = 1$) the optimization problem as defined in (C.26) simplifies to:

$$\max_{\{c_{O,t}\}_{t=1}^{\infty}, \{c_{Y,t}\}_{t=0}^{\infty}} U_{O,0}$$

Writing $U_{O,0}$ out explicitely:

$$U_{O,0} = u(c_{O,0}) + \theta u(w - c_{O,0} - c_{Y,0}) + \lambda U_{Y,0}$$

Now we can substitute in subsequently $U_{Y,t}$ and $U_{O,t+1}$ for all $t \geq 0$:

$$\begin{aligned} U_{O,0} &= u(c_{O,0}) + \theta u(w - c_{O,0} - c_{Y,0}) + \lambda \left[ u(c_{Y,0} + \theta u(w - c_{O,0} - c_{Y,0}) \right. \\ &\left. + \beta \left[ u(c_{O,1}) + \theta u(w - c_{O,1} - c_{Y,1}) + \lambda \left[ u(c_{Y,1}) + \theta u(w - c_{O,1} - c_{Y,1}) + ... \right]\right]\right] \end{aligned}$$

Grouping the terms per time period we can find the infinite sum representation:

$$\begin{aligned} U_{O,0} = \sum_{t=0}^{\infty} (\beta\lambda)^t \left( u(c_{O,t}) + \theta u(w - c_{O,t} - c_{Y,t}) + \lambda \left( u(c_{Y,t}) \right.\right. \\ \left.\left. + \theta u(w - c_{O,t} - c_{Y,t}) \right)\right) \end{aligned}$$

Therefore we can write the maximization problem when only the old decide in the following way:

$$\max_{\{c_{O,t}\}_{t=1}^{\infty}, \{c_{Y,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta\lambda)^t \left( u(c_{O,t}) + \theta(1+\lambda) u(w - c_{O,t} - c_{Y,t}) + \lambda u(c_{Y,t}) \right)$$

From this equation it is easily seen that there exists a recursive formulation, where consumption of the old today or the level of government debt, $c_O$, used as a state variable. It is given by the following Bellman equation:

$$\begin{aligned} V_O^{Comm}(c_O) &= \max_{c_Y, c'_O} u(c_O) + \theta(1+\lambda)u(w - c_O - c_Y) + \lambda\left[u(c_Y)\right. \\ &\quad \left. + \beta V_O^{Comm}(c'_O)\right] \end{aligned} \tag{C.28}$$

Similarly, if I did not substitute for $g$ equation (C.9) would arise.

**Only the Old Decide: First Order Conditions and Solutions**

The first order conditions of problem (C.28) are given by:

$$-\theta(1+\lambda)u'(w - c_O - c_Y) + \lambda u'(c_Y) = 0 \tag{C.29}$$

$$\lambda\beta \frac{\partial V_O^{Comm}(c'_O)}{\partial c'_O} = 0 \tag{C.30}$$

The envelope theorem yields:

$$\lambda\beta \frac{\partial V_O^{Comm}(c'_O)}{\partial c'_O} = \lambda\beta[u'(c'_O) - \theta(1+\lambda)u'(w - c'_O - c'_Y)] = 0$$

Suppose we have CES-utility as defined in (C.3). Then equation (C.29) and (C.30) are given by:

$$-\theta(1+\lambda)(w - c_O - c_Y)^{-\sigma} + \lambda(c_Y)^{-\sigma} = 0 \tag{C.31}$$

$$(c'_O)^{-\sigma} - \theta(1+\lambda)(w - c'_O - c'_Y)^{-\sigma} = 0 \text{ for } \lambda, \beta > 0 \tag{C.32}$$

where I have used the envelope condition to substitute out $\frac{\partial V_O^{Comm}(c'_O)}{\partial c'_O}$.

It is easy to show that solving the equation system consisting of equation

(C.31) and (C.32) for $c_Y$ and $c'_O$ yields the following solutions:

$$c_Y = \frac{\lambda^{\frac{1}{\sigma}}}{\left((\theta(1+\lambda))^{\frac{1}{\sigma}} + \lambda^{\frac{1}{\sigma}}\right)}(w - c_O) \tag{C.33}$$

$$c'_O = b = \frac{1}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}} + 1 + \lambda^{\frac{1}{\sigma}}\right]} w \text{for } \lambda, \beta > 0 \tag{C.34}$$

As we can see from these equations consumption of the old is constant from period 1 onwards. Consumption of the young depends on the state variable and thus is constant from period 1 onwards as well, but different in period 0 depending on the initial debt level $c_{O,0}$:

$$c_{Y,t} = \begin{cases} \frac{\lambda^{\frac{1}{\sigma}}}{\left((\theta(1+\lambda))^{\frac{1}{\sigma}} + \lambda^{\frac{1}{\sigma}}\right)}(w - c_{O,t}) & \text{in period } t = 0 \\ \frac{\lambda^{\frac{1}{\sigma}}}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}} + 1 + \lambda^{\frac{1}{\sigma}}\right]} w & \text{in period } t > 0 \end{cases} \tag{C.35}$$

**Only the Young Decide: Recursive Formulation of the Maximization Problem**

Similarly to the case when only the old decide one can use the infinite sum representation to write down the maximization problem when only the young decide:

$$\max_{\{c_{O,t}\}_{t=1}^{\infty}, \{c_{Y,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta\lambda)^t \left(u(c_{Y,t}) + \theta u(w - c_{O,t} - c_{Y,t}) + \beta\left(u(c_{O,t+1}) + \theta u(w - c_{O,t+1} - c_{Y,t+1})\right)\right)$$

It is obvious that from period 1 onwards both maximization problems are virtually the same as in period 1 the young today are old themselves. The problem can thus be formulated in two stages using the same definition of the value function found in the case of the old:

$$\begin{aligned}
\{c_{Y,0}, c_{O,1}\} &= \arg\max \left\{u(c_{Y,0}) + \theta u(w - c_{O,0} - c_{Y,0}) + V_O^{Comm}(c_{O,1})\right\} \\
V_O^{Comm}(c_O) &= \max_{c_Y, c'_O} u(c_O) + \theta(1+\lambda)u(w - c_O - c_Y) + \\
&\quad \lambda\left[u(c_Y) + \beta V_O^{Comm}(c'_O)\right] \quad \text{for all } t > 0
\end{aligned}$$

This problem can be solved by backwards induction. In the second stage the young maximize their utility in old age choosing a plan for $c_Y$ and $c'_O$ for all $t > 0$ which is similar to the problem where only the old decide. In the first stage they maximize their utility from private and public consumption and the value of their old age utility with respect to $c_{Y,0}$ and $c_{O,1}$ given the future plans for $c_Y$ and $c'_O$. The equivalent formulation without substituting for $g$ is given in equation (C.12).

### Only the Young Decide: First Order Conditions and Solutions

The first order conditions of this problem for periods $t > 0$ were already derived above (see equation (C.31) and (C.32)). In $t = 0$ the first order condition are given by:

$$\begin{aligned}
u'(c_{Y,0}) - \theta u'(w - c_{O,0} - c_{Y,0}) &= 0 \\
\frac{\partial V_O^{Comm}(c_{O,1})}{\partial c_{O,1}} &= 0
\end{aligned}$$

The envelope theorem yields:

$$\frac{\partial V_O^{Comm}(c_{O,1})}{\partial c_{O,1}} = u'(c_{O,1}) - \theta(1+\lambda)u'(w - c_{O,1} - c_{Y,1})$$

With CES-utility (and making use of the envelope condition):

$$\begin{aligned}
(c_{Y,0})^{-\sigma} - \theta(w - c_{O,0} - c_{Y,0})^{-\sigma} &= 0 \\
(c_{O,1})^{-\sigma} - \theta(1+\lambda)(w - c_{O,1} - c_{Y,1})^{-\sigma} &= 0
\end{aligned}$$

Solving this system of equations for $c_{Y,0}$ and $c_{O,1}$ (also using equation (C.31) to substitute for $c_{Y,1}$) yields the solutions in terms of the state variable $c_{O,0}$:

$$\begin{aligned} c_{Y,0} &= \frac{1}{\left(1+\theta^{\frac{1}{\sigma}}\right)}(w-c_{O,0}) \\ c_{O,1} &= b = \frac{1}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}}+1+\lambda^{\frac{1}{\sigma}}\right]}w \end{aligned}$$

For $t>0$ the solutions for $c'_O$ and $c_Y$ are of course the same as for the case where only the old decide (see equation (C.34) for consumption in old age and equation (C.35) for consumption in youth).

### General Commitment Case: Recursive Formulation of the Maximization Problem

Having found the recursive formulations of the extreme cases ($\omega=1$ and $\omega=0$), we can easily find a recursive formulation for the general commitment case as a combination of those two extreme cases. First transform the maximization object from the problem defined in equation (C.26) as a weighted sum of the two infinite sum representations:

$$\begin{aligned} & (1-\omega)U_{Y,0}+\omega U_{O,0} \\ = \ & (1-\omega)\sum_{t=0}^{\infty}(\beta\lambda)^t\left(u(c_{Y,t})+\theta u(w-c_{O,t}-c_{Y,t})+\beta\left(u(c_{O,t+1})\right.\right. \\ & \left.\left.+\theta u(w-c_{O,t+1}-c_{Y,t+1})\right)\right) \\ & +\omega\sum_{t=0}^{\infty}(\beta\lambda)^t\left(u(c_{O,t})+\theta(1+\lambda)u(w-c_{O,t}-c_{Y,t})+\lambda u(c_{Y,t})\right) \\ = \ & (1-\omega)\left(u(c_{Y,0})+\theta u(w-c_{O,0}-c_{Y,0})\right)+\omega\left(u(c_{O,0})+\theta u(w-c_{O,0}-c_{Y,0})\right) \\ & +\beta(1-\omega+\omega\lambda)\sum_{t=1}^{\infty}(\beta\lambda)^{t-1}\left(u(c_{O,t})+\theta(1+\lambda)u(w-c_{O,t}-c_{Y,t})+\lambda u(c_{Y,t})\right) \end{aligned}$$

One can now express the problem of maximizing this object, analogously as in the case where only the young decide, as a two stage recursive problem with state variable $c_O$:

$$\begin{aligned}\{c_{Y,0}, c_{O,1}\} &= \arg\max\left\{(1-\omega)q(c_{Y,0}, c_{O,0}) + \omega v(c_{Y,0}, c_{O,0})\right.\\ &\quad \left.+\beta\widetilde{\omega}V_O^{Comm}(c_{O,1})\right\}\\ V_O^{Comm}(c_O) &= \max_{c_Y, c'_O} v(c_Y, c_O) + \lambda\beta V_O^{Comm}(c'_O) \text{ for all } t > 0\end{aligned}$$

where $q(.)$, $v(.)$ and $\widetilde{\omega}$ are defined as follows to make the equation more readable:

$$\begin{aligned}&q(c_{Y,0}, c_{O,0}) = u(c_{Y,0}) + \theta u(w - c_{O,0} - c_{Y,0})\\ &v(c_{Y,0}, c_{O,0}) = u(c_{O,0}) + (1+\lambda)\theta u(w - c_{O,0} - c_{Y,0}) + \lambda u(c_{Y,0})\\ &\widetilde{\omega} = (1 - \omega + \omega\lambda)\end{aligned}$$

Similar as in the case where only the young decide, the problem can be solved by backwards induction. First one has to solve the second stage problem (equivalent to case where only the old decide) and then the first stage given results from the second stage. The equivalent formulation without substituting for $g$ is given in equation (C.14).

#### General Commitment Case: First Order Conditions and Solutions

Given the two distinct stages first order conditions in period 0 will differ from subsequent first order conditions. Obviously in subsequent periods the first order conditions will be equal to the ones of the case where only the old decide (see equation (C.31) and (C.32)). In period 0 the first order conditions are given by:

$$\begin{aligned}\widetilde{\omega}u'(c_{Y,0}) - \theta\left(1+\omega\lambda\right)u'(w - c_{O,0} - c_{Y,0}) &= 0\\ \beta\widetilde{\omega}\frac{\partial V_O^{Comm}(c_{O,1})}{\partial c_{O,1}} &= 0\end{aligned}$$

The envelope theorem yields:

$$\frac{\partial V_O^{Comm}(c_{O,1})}{\partial c_{O,1}} = u'(c_{O,1}) - \theta(1+\lambda)u'(w - c_{O,1} - c_{Y,1})$$

With CES-utility (and making use of the envelope condition):

$$\begin{aligned}
\widetilde{\omega}(c_{Y,0})^{-\sigma} - \theta\left(1+\omega\lambda\right)\left(w - c_{O,0} - c_{Y,0}\right)^{-\sigma} &= 0 \\
\beta\widetilde{\omega}[(c_{O,1})^{-\sigma} - \theta(1+\lambda)(w - c_{O,1} - c_{Y,1})^{-\sigma}] &= 0
\end{aligned}$$

Solving this system of equations for $c_{Y,0}$ and $c_{O,1}$:

$$\begin{aligned}
c_{Y,0} &= \frac{\widetilde{\omega}^{\frac{1}{\sigma}}}{[\theta(1+\omega\lambda)]^{\frac{1}{\sigma}} + \widetilde{\omega}^{\frac{1}{\sigma}}}(w - c_{O,0}) \\
c_{O,1} &= \frac{1}{\left[(\theta(1+\lambda))^{\frac{1}{\sigma}} + 1 + \lambda^{\frac{1}{\sigma}}\right]} w \text{ for } \beta > 0, \widetilde{\omega} > 0
\end{aligned} \tag{C.36}$$

Note that:

$$\beta > 0 \cap \widetilde{\omega} > 0 \Longleftrightarrow \beta > 0 \cap (\omega < 1 \cup \lambda > 0)$$

This condition captures the fact that the demand for bonds in period $t = 0$ is only determined if either the young have some decision power or there is some altruism. Of course the solutions for periods $t > 0$ are again equal to the case where only the old decide (see equation (C.34) for consumption in old age and equation (C.35) for consumption in youth).

**Proof: Private Consumption in Period 0 is Higher the Higher the Power of the Young**

To show this I calculate the partial derivative of the solution for consumption of the young in period 0 with respect to the power of the old $\omega$ and show that it is negative. As $(1-\omega)$ is the power of the young, it must then be the case that the partial derivative with respect to this parameter is positive. First write $c_{Y,0}$ as a combination of two functions of $\omega$ (and other parameters):

$$\begin{aligned} c_{Y,0}(f(\omega,\lambda),h(\omega,\lambda,\theta)) &= \frac{f(\omega,\lambda)^{\frac{1}{\sigma}}}{h(\omega,\lambda,\theta)^{\frac{1}{\sigma}}+f(\omega,\lambda)^{\frac{1}{\sigma}}}(w-c_{O,0}) \\ &= \frac{(1-\omega+\omega\lambda)^{\frac{1}{\sigma}}}{[\theta(1+\omega\lambda)]^{\frac{1}{\sigma}}+(1-\omega+\omega\lambda)^{\frac{1}{\sigma}}}(w-c_{O,0}) \end{aligned}$$

Then calculate the partial derivative with respect to $\omega$ (omitting functional arguments for simplicity):

$$\begin{aligned} \frac{\partial c_{Y,0}(f,h)}{\partial\omega} &= \frac{\frac{1}{\sigma}f^{\frac{1}{\sigma}-1}\frac{\partial f}{\partial\omega}\left[h^{\frac{1}{\sigma}}+f^{\frac{1}{\sigma}}\right]-f^{\frac{1}{\sigma}}\left[\frac{1}{\sigma}h^{\frac{1}{\sigma}-1}\frac{\partial h}{\partial\omega}+\frac{1}{\sigma}f^{\frac{1}{\sigma}-1}\frac{\partial f}{\partial\omega}\right]}{\left[h^{\frac{1}{\sigma}}+f^{\frac{1}{\sigma}}\right]^2}(w-c_{O,0}) \\ &= \frac{\frac{1}{\sigma}f^{\frac{1}{\sigma}}h^{\frac{1}{\sigma}}\left[\frac{\partial f}{\partial\omega}f^{-1}-\frac{\partial h}{\partial\omega}h^{-1}\right]}{\left[h^{\frac{1}{\sigma}}+f^{\frac{1}{\sigma}}\right]^2}(w-c_{O,0}) \end{aligned}$$

As $w$ is the maximum debt level (as discussed above) it follows that $w-c_{O,0}\geq 0$. The square in the denominator of the other factor must be positive, so that I can concentrate on the numerator. It is clear that because of $\sigma,\theta>0$, $0<\lambda<1$ and $0<\omega<1$ it must hold that $f>0$, $h>0$ and thus $\frac{1}{\sigma}>0$,$f^{\frac{1}{\sigma}}>0$ and $h^{\frac{1}{\sigma}}>0$. I thus concentrate on the square bracket in the numerator:

$$\frac{\partial f}{\partial\omega}f^{-1}-\frac{\partial h}{\partial\omega}h^{-1}=-\frac{(1-\lambda)}{f}-\frac{\theta\lambda}{h}<0$$

■

### C.6.2 The Political Equilibrium

**Proof of Proposition 1**

Substituting for the government budget constraint in each period the maximization problem defined in Definition C.5 can be written out as:

$$\begin{aligned} \max_{c_Y, c'_O} \quad & (1-\omega)U_Y(\mathbf{c}_Y, \mathbf{c}_O) + \omega U_O(\mathbf{c}_Y, \mathbf{c}_O) \\ \text{where} \quad & U_Y(\mathbf{c}_Y, \mathbf{c}_O) = u(c_Y) + \theta u(w - c_Y - c_O) + \beta U_O(\mathbf{c}'_Y, \mathbf{c}'_O) \\ & U_O(\mathbf{c}_Y, \mathbf{c}_O) = u(c_O) + \theta u(w - c_Y - c_O) + \lambda U_Y(\mathbf{c}'_Y, \mathbf{c}'_O) \end{aligned}$$

Now let $C^n_{i=O,Y}(c_O)$ denote the n-th period reaction after the state variable $c_O$ has occurred, such that for example $C^0_O(c_O) = c_O$, $C^1_O(c_O) = C_O(c_O) = c'_O$, $C^2_O(c_O) = C_O(C_O(c_O))$, etc. Using this notation we can rewrite the optimization problem defined in Definition C.5 as follows:

$$\begin{aligned} \max_{c_Y, c'_O} \quad & (1-\omega)\left\{u(c_Y) + \theta u(w - c_Y - c_O) + \beta\left[u(c'_O)\right.\right. \\ & +\theta(1+\lambda)u(w - C_Y(c'_O) - c'_O) + \lambda u(C_Y(c'_O)) \\ & + \sum_{i=1}^{\infty} \beta^i \lambda^{i-1}\left[u(C^i_O(c'_O)) + \theta(1+\lambda)u(w - C^{i+1}_Y(c'_O) - C^i_O(c'_O))\right. \\ & \left.\left.\left. +\lambda u(C^{i+1}_Y(c'_O))\right]\right]\right\} \\ & +\omega\left\{u(c_O) + \theta(1+\lambda)u(w - c_Y - c_O) + \lambda u(c_Y) + \lambda\beta\left[u(c'_O)\right.\right. \\ & +\theta(1+\lambda)u(w - C_Y(c'_O) - c'_O) + \lambda u(C_Y(c'_O)) \\ & + \sum_{i=1}^{\infty} \beta^i \lambda^{i-1}\left[u(C^i_O(c'_O))\right. \\ & \left.\left.\left. +\theta(1+\lambda)u(w - C^{i+1}_Y(c'_O) - C^i_O(c'_O)) + \lambda u(C^{i+1}_Y(c'_O))\right]\right]\right\} \end{aligned}$$

Realizing that similar to the commitment case, the old and the young have virtually the same preferences from the second period onwards, the expression can be simplified to:

$$\begin{aligned}
\max_{c_Y, c'_O} \quad & (1-\omega)q(c_Y, c_O) + \omega v(c_Y, c_O) + \beta\widetilde{\omega}\left[u(c'_O)\right. \\
& +\theta(1+\lambda)u(w - C_Y(c'_O) - c'_O) \\
& +\lambda u(C_Y(c'_O)) + \sum_{i=1}^{\infty} \beta^i \lambda^{i-1} \left[u(C_O^i(c'_O))\right. \\
& +\theta(1+\lambda)u(w - C_Y^{i+1}(c'_O) - C_O^i(c'_O)) \\
& \left.\left. +\lambda u(C_Y^{i+1}(c'_O))\right]\right]
\end{aligned}$$

Similarly as for the commitment case it is now possible to find a "two-stage" recursive formulation:

$$\begin{aligned}
& < C_O(c_O), C_Y(c_O) >= \arg\max_{c_Y, c'_O} (1-\omega)q(c_Y, c_O) + \omega v(c_Y, c_O) + \beta\widetilde{\omega} V_O(c'_O) \\
& V_O(c'_O) = v(C_Y(c'_O), c'_O) + \beta\lambda V_O(C_O(c'_O))
\end{aligned}$$

where $v(.)$, $q(.)$ and $\widetilde{\omega}$ are defined as above. However note that the problem does not have two stages in the usual sense. Each generation only solves one singular maximization problem given the actions of future generations. Therefore there is no maximization operator in the definition of $V_O$. However as each generation is symmetric given the state variable, it is clear that policy functions are the same for each generation given the state variable. One can use this fact and the system of equations arising from the first order conditions to find the Markov Perfect Equilibrium. First calculate the first order conditions for some generation:

$$(1-\omega)\frac{\partial q(c_Y, c_O)}{\partial c_Y} + \omega\frac{\partial v(c_Y, c_O)}{\partial c_Y} = 0 \quad \text{(C.37)}$$

$$\beta\widetilde{\omega}\frac{\partial V_O(c'_O)}{\partial c'_O} = 0 \quad \text{(C.38)}$$

Note that equation (C.37) can directly be solved to yield $c_Y = C_Y(c_O)$ as it only depends on contemporaneous values. Suppose CES-utility, then equation (C.37) is given by:

$$\begin{aligned}
& (1-\omega)\frac{\partial q(c_Y, c_O)}{\partial c_Y} + \omega\frac{\partial v(c_Y, c_O)}{\partial c_Y} = 0 \\
\Longleftrightarrow\quad & (1-\omega)\left[u'(c_Y) - \theta u'(w - c_O - c_Y)\right] + \omega\left[-(1+\lambda)\theta u'(w - c_O - c_Y)\right. \\
& \left.+\lambda u'(c_Y)\right] = 0 \\
\Longleftrightarrow\quad & (1-\omega+\omega\lambda)u'(c_Y) - (1+\omega\lambda)\theta u'(w - c_O - c_Y) = 0 \\
\Longleftrightarrow\quad & (1-\omega+\omega\lambda)(c_Y)^{-\sigma} - (1+\omega\lambda)\theta(w - c_O - c_Y)^{-\sigma} = 0
\end{aligned}$$

Solving this equation for $c_Y$ yields:

$$\begin{aligned}
c_Y &= C_Y(c_O) = \frac{1}{1+\xi}(w - c_O) \qquad \text{(C.39)} \\
\text{where } \xi &= \left(\frac{[\theta(1+\omega\lambda)]}{\widetilde{\omega}}\right)^{\frac{1}{\sigma}}
\end{aligned}$$

To find the solution for government debt first calculate the partial derivative $\frac{\partial V_O(c'_O)}{\partial c'_O}$ using the definition of $V_O$ (omitting functional arguments of reaction functions for simplicity):

$$\frac{\partial V_O(c'_O)}{\partial c'_O} = \frac{\partial v\left(C_Y, c'_O\right)}{\partial c'_O} + \frac{\partial v\left(C_Y, c'_O\right)}{\partial C_Y}\frac{\partial C_Y}{\partial c'_O} + \beta\lambda\frac{\partial V_O(C_O)}{\partial C_O}\frac{\partial C_O}{c'_O} \qquad \text{(C.40)}$$

Iterating forward by one period the first order condition for consumption of the old, equation (C.38), leads to an envelope condition:

$$\frac{\partial V_O(c''_O)}{\partial c''_O} = \frac{\partial V_O(C_O)}{\partial C_O} = 0 \text{ for } \beta, \tilde{\omega} > 0$$

Now use this envelope condition and equation (C.40):

$$\begin{aligned}
\beta\widetilde{\omega}\frac{\partial V_O(c'_O)}{\partial c'_O} &= \beta\widetilde{\omega}\left[\frac{\partial v\left(C_Y, c'_O\right)}{\partial c'_O} + \frac{\partial v\left(C_Y, c'_O\right)}{\partial C_Y}\frac{\partial C_Y}{\partial c'_O}\right] \\
&= \beta\widetilde{\omega}\left[u'(c'_O) - (1+\lambda)\theta u'(w - c'_O - C_Y)\left[1 + C'_Y\right]\right. \\
&\quad \left.+\lambda u'(C_Y)C'_Y\right] = 0
\end{aligned}$$

Now suppose again CES-utility. Conveniently, the policy function $C_Y(c'_O)$ and its derivative $C'_Y(c'_O)$ are already known (using equation (C.39)) so that they can be substituted in directly:

$$\beta\widetilde{\omega}\left[c_O'^{-\sigma} - (1+\lambda)\theta(w-c'_O)^{-\sigma}\left[1-\frac{1}{1+\xi}\right]^{1-\sigma} - \lambda(w-c'_O)^{-\sigma}\left(\frac{1}{1+\xi}\right)^{1-\sigma}\right] = 0$$

Solving this equation for $c'_O$ yields:

$$c'_O = \frac{(1+\xi)^{\frac{1-\sigma}{\sigma}}}{(1+\xi)^{\frac{1-\sigma}{\sigma}} + [(1+\lambda)\theta\xi^{1-\sigma}+\lambda]^{\frac{1}{\sigma}}} w \text{ for } \beta, \widetilde{\omega} > 0 \qquad \text{(C.41)}$$

Analogously as for the commitment case, the new government debt issues are not always determined. As shown above $\beta > 0 \cap \widetilde{\omega} > 0$ implies $\beta > 0 \cap (\omega < 1 \cup \lambda > 0)$, which captures the fact that government debt is only determined if either the young have some decision power or there is some altruism. The reason is as was already discussed that the old do not care directly how much government debt there is.

The policy function for the public good can be calculated using the government budget constraint:

$$\begin{aligned} G(c_O) &= w - C_Y(c_O) - c_O \\ &= \frac{\xi}{1+\xi}(w - c_O) \end{aligned}$$

■

**Policy Function for the Tax Rate**

The implied policy function for the tax rate (denote it by $T(c_O)$) can be calculated by inserting the policy functions for consumption into equation

(C.27):

$$
\begin{aligned}
T(c_O)w &= w - C_Y(c_O) - \beta\frac{u'(C_O(c_O))}{u'(C_Y(c_O))}C_O(c_O) \\
&= w - \frac{1}{1+\xi}(w-c_O) - \beta\frac{\left[\frac{(1+\xi)^{\frac{1-\sigma}{\sigma}}}{(1+\xi)^{\frac{1-\sigma}{\sigma}}+[(1+\lambda)\theta\xi^{1-\sigma}+\lambda]^{\frac{1}{\sigma}}}w\right]^{1-\sigma}}{\left[\frac{1}{1+\xi}(w-c_O)\right]^{-\sigma}} \\
\Longleftrightarrow T(c_O) &= 1 - \frac{1}{1+\xi}\frac{(w-c_O)}{w} - \beta\left[\frac{(1+\xi)^{\frac{1-\sigma}{\sigma}}}{(1+\xi)^{\frac{1-\sigma}{\sigma}}+[(1+\lambda)\theta\xi^{1-\sigma}+\lambda]^{\frac{1}{\sigma}}}\right]^{1-\sigma} \\
&\quad \left[\frac{1}{1+\xi}\frac{(w-c_O)}{w}\right]^{\sigma}
\end{aligned}
$$

**Proof of Proposition 2**

Consider the following formulation for the level of government debt in the political equilibrium (PE) and under commitment (Comm):

$$
\begin{aligned}
c_O^{\prime Comm} &= \frac{1}{\left(\frac{\theta(1+\lambda)\hat{\xi}^{1-\sigma}+\lambda}{(1+\hat{\xi})^{1-\sigma}}\right)^{\frac{1}{\sigma}}+1} \\
c_O^{\prime PE} &= \frac{1}{\left(\frac{\theta(1+\lambda)\xi^{1-\sigma}+\lambda}{(1+\xi)^{1-\sigma}}\right)^{\frac{1}{\sigma}}+1}
\end{aligned}
$$

where $\hat{\xi} = \left(\frac{(1+\lambda)\theta}{\lambda}\right)^{\frac{1}{\sigma}}$. Note that now the two debt levels look very similar, which has been achieved by setting $\hat{\xi}$ in a certain way. In fact, we can set $\omega = 1$ in the political equilibrium to turn it into the commitment solution. This would transform $\xi$ directly into $\hat{\xi}$. Now define the following function $f(x)$:

$$
f(x) = \left(\frac{\theta(1+\lambda)x^{1-\sigma}+\lambda}{(1+x)^{1-\sigma}}\right)^{\frac{1}{\sigma}}
$$

Note that one can write the debt levels in terms of this function $f$ at different realizations for the function argument: $x = \xi$ for the political equilibrium and $x = \hat{\xi}$ for the commitment case:

$$\begin{aligned} c_O'^{Comm} &= \frac{1}{f(\hat{\xi}) + 1} \\ c_O'^{PE} &= \frac{1}{f(\xi) + 1} \end{aligned}$$

The proof now consists of two parts. First, it can be shown that $f'(x) < 0$ for $\frac{1}{\sigma} < 1$ and $f'(x) > 0$ for $\frac{1}{\sigma} > 1$. Simply differentiate the function $f(.)$ with respect to its argument:

$$\begin{aligned} \frac{df(x)}{dx} &= \left(\frac{1}{\sigma} - 1\right)\left(\frac{\theta(1+\lambda)x^{1-\sigma} + \lambda}{(1+x)^{1-\sigma}}\right)^{\frac{1}{\sigma}-1} \\ &\quad \frac{\theta(1+\lambda)x^{-\sigma} + [\theta(1+\lambda)x^{1-\sigma} + \lambda](1+x)^{-1}}{(1+x)^{1-\sigma}} \end{aligned}$$

It is clear that the sign of this derivative is negative in the case of $\frac{1}{\sigma} < 1$ and positive in the case of $\frac{1}{\sigma} > 1$ for all $0 < \omega < 1$, $\theta, \lambda > 0$. For the knife-edge case of $\frac{1}{\sigma} = 1$ the derivative is zero. This means that in this case the value of $x$ is irrelevant as $f(x)$ is constant in $x$ (which can also be seen by introspection). Thus for $\frac{1}{\sigma} = 1$ the commitment solution is equal to the political equilibrium.

Second, it can be shown that $\hat{\xi} > \xi$ for $\omega < 1$ and $\lambda \leq 1$:

$$\begin{aligned} &\hat{\xi} = \left(\frac{(1+\lambda)\theta}{\lambda}\right)^{\frac{1}{\sigma}} > \left(\frac{(1+\omega\lambda)\theta}{(1-\omega+\omega\lambda}\right)^{\frac{1}{\sigma}} = \xi \\ &\Leftrightarrow \frac{1+\lambda}{\lambda} > \frac{1+\omega\lambda}{1-\omega+\omega\lambda} \\ &\Leftrightarrow (1+\lambda)(1-\omega+\omega\lambda) > \lambda(1+\omega\lambda) \\ &\Leftrightarrow ((1+\lambda) > (1+\omega\lambda)) \cap ((1-\omega) + \omega\lambda \geq \lambda = (1-\omega)\lambda + \omega\lambda) \end{aligned}$$

Of course it is easy to see that for $\omega = 1$ $\hat{\xi} = \xi$ for all $\lambda, \theta$.

Thus it has been shown that for all interesting parameter constellations $\hat{\xi} > \xi$. Combined with the previous finding that $f'(x) < 0$ for $\frac{1}{\sigma} < 1$ and $f'(x) > 0$

for $\frac{1}{\sigma} > 1$, it is easy to see that the following must hold:

$$\begin{array}{lllll} c_O^{\prime Comm} = \dfrac{1}{f(\hat{\xi})+1} & < & c_O^{\prime PE} = \dfrac{1}{f(\xi)+1} & \text{for } \dfrac{1}{\sigma} > 1 \\ c_O^{\prime Comm} = \dfrac{1}{f(\hat{\xi})+1} & > & c_O^{\prime PE} = \dfrac{1}{f(\xi)+1} & \text{for } \dfrac{1}{\sigma} < 1 \end{array}$$

This completes the proof.■

**Comparative Statics: The Effect of the Political Power of Young and Old**

Define a function $h(.)$ as follows:

$$c_O^{\prime PE} = \frac{1}{h(\xi(\theta,\omega,\lambda),\theta,\lambda)+1}$$

where $\xi(\theta,\omega,\lambda) = \left(\frac{(1+\omega\lambda)\theta}{(1-\omega+\omega\lambda}\right)^{\frac{1}{\sigma}}$ and $h(\xi(\theta,\omega,\lambda),\theta,\lambda) = \left(\frac{\theta(1+\lambda)\xi(.)^{1-\sigma}+\lambda}{(1+\xi(.))^{1-\sigma}}\right)^{\frac{1}{\sigma}}$. This function $h(.)$ is similar to the function $f(.)$ defined in the proof of Proposition 2 except that it takes the parameters explicitely as arguments. Differentiating $h(.)$ with respect to $\omega$:

$$\frac{dh(.)}{d\omega} = \frac{\partial h(.)}{\partial \xi}\frac{\partial \xi(.)}{\partial \omega}$$

It is easy to show that $\frac{\partial \xi(.)}{\partial \omega} > 0$ must always hold:

$$\frac{\partial \xi(.)}{\partial \omega} = \frac{1}{\sigma}\left(\frac{\theta(1+\omega\lambda)}{1-\omega+\omega\lambda}\right)^{\frac{1}{\sigma}-1}\frac{\theta}{(1-\omega+\omega\lambda)^2} > 0$$

For $\frac{1}{\sigma} < 1$ we know from Proposition 2 that $\frac{\partial h(.)}{\xi} < 0$ and for $\frac{1}{\sigma} > 1$ $\frac{\partial h(.)}{\xi} > 0$. From introspection it can be seen that $\frac{\partial c_O^{\prime PE}}{\partial h} < 0$. Thus the comparative statics effect of $\omega$ is given by:

$$\frac{dc_O^{\prime PE}}{d\omega} = \frac{\partial c_O^{\prime PE}}{\partial h}\frac{\partial h(.)}{\partial \xi}\frac{\partial \xi(.)}{\partial \omega}\begin{cases} > 0 \text{ for } \frac{1}{\sigma} < 1 \\ < 0 \text{ for } \frac{1}{\sigma} > 1 \end{cases}$$

**Comparative Statics: The Effect of the Preference for Public Goods**

Define the functions $h(.)$ and $\xi(.)$ as above. Differentiating $h(.)$ partially with respect to $\theta$ yields:

$$\frac{\partial h(.)}{\partial \theta} = \frac{1}{\sigma}\left(\frac{(1+\lambda)\theta\xi^{1-\sigma}+\lambda}{(1+\xi)^{1-\sigma}}\right)^{\frac{1}{\sigma}-1}\frac{(1+\lambda)\xi^{1-\sigma}}{(1+\xi)^{1-\sigma}} > 0$$

The partial derivative of $\xi(.)$ with respect to $\theta$ is given by:

$$\frac{\partial \xi(.)}{\partial \theta} = \frac{1}{\sigma}\left(\frac{\theta(1+\omega\lambda)}{1-\omega+\omega\lambda}\right)^{\frac{1}{\omega}-1}\frac{1+\omega\lambda}{(1-\omega+\omega\lambda)} > 0$$

The total comparative statics effect of $\theta$ can thus be stated as follows:

$$\frac{dc_O'^{PE}}{d\theta} = \frac{\partial c_O'^{PE}}{\partial h}\left(\frac{\partial h(.)}{\partial \xi}\frac{\partial \xi(.)}{\partial \theta}+\frac{\partial h(.)}{\partial \theta}\right)\begin{cases}\text{ambiguous} & \text{for } \frac{1}{\sigma}<1\\ <0 & \text{for } \frac{1}{\sigma}>1\end{cases}$$

**Comparative Statics: The Effect of Altruism**

Define again the functions $h(.)$ and $\xi(.)$ as above. Differentiating $h(.)$ partially with respect to $\lambda$ yields:

$$\frac{\partial h(.)}{\partial \lambda} = \frac{1}{\sigma}\left(\frac{(1+\lambda)\theta\xi^{1-\sigma}+\lambda}{(1+\xi)^{1-\sigma}}\right)^{\frac{1}{\sigma}-1}\frac{\theta\xi^{1-\sigma}+1}{(1+\xi)^{1-\sigma}} > 0$$

The partial derivative of $\xi(.)$ with respect to $\theta$ is given by:

$$\frac{\partial \xi(.)}{\partial \lambda} = \frac{1}{\sigma}\left(\frac{\theta(1+\omega\lambda)}{1-\omega+\omega\lambda}\right)^{\frac{1}{\omega}-1}\frac{-\theta\omega^2}{(1-\omega+\omega\lambda)^2} < 0$$

The total comparative statics effect of $\theta$ can thus be stated as follows:

$$\frac{dc_O'^{PE}}{d\lambda} = \frac{\partial c_O'^{PE}}{\partial h}\left(\frac{\partial h(.)}{\partial \xi}\frac{\partial \xi(.)}{\partial \lambda}+\frac{\partial h(.)}{\partial \lambda}\right)\begin{cases}<0 & \text{for } \frac{1}{\sigma}<1\\ \text{ambiguous} & \text{for } \frac{1}{\sigma}>1\end{cases}$$

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